Mathematics 1B Professor K. A. Ribet Spring 1990

First Midterm Exam—60 points

1a *(5 points).* Find $\lim_{t\to 0} \dfrac{\cos 3t - 1}{\cos 4t - 1}$.

1b *(7 points).* Calculate $\displaystyle\int_2^4 \frac{\sqrt{16-x^2}}{x^2}\,dx$.

2a *(6 points).* Find $\displaystyle\int \frac{\ln(x^2)}{x^2}\,dx$.

2b *(6 points).* What approximation to $\displaystyle\int_0^6 (x^2 - 2x - 6)\,dx$ is furnished by Simpson's Rule, when the interval $[0, 6]$ is divided into 6 equal pieces?

[In problems 3–4, do *not* evaluate the integrals!]

3 *(8 points).* The region between $y = \sin x$ and the x-axis, from $x = 0$ to $x = \pi/2$, is covered with a thin wafer weighing 20 pounds per unit area. Express as a definite integral the wafer's moment of inertia about the line $y = -3$.

4 *(6 points).* A thin uniform wire weighing 300 tons is fitted over that part of the curve $y = x^3$ which runs from $x = 1$ to $x = 2$. Express in terms of definite integrals the x- and y-coordinates of the centroid of the wire.

5a *(5 points).* Evaluate $\lim_{t\to 1} \dfrac{t-1}{\sqrt{t+1} - \sqrt{t-1}}$.

5b *(8 points).* Find A, B, and C: $\dfrac{x^2 - 2x + 4}{(x-1)(x^2 - x + 1)} = \dfrac{A}{x-1} + \dfrac{Bx + C}{x^2 - x + 1}$.

6 *(9 points).* Calculate $\displaystyle\int_0^{1/2} \frac{8 - 16x}{8x^2 - 4x + 1}\,dx$.