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Question 1: [20 points]

Suppose you have a new element called Dibblium (Db). Through careful measurements you determine that Db has the following properties:

Heat capacity C_p of gas	1.17 (kJ/kg-K)
Heat capacity C_p of liquid	4.19 (kJ/kg-K)
Heat capacity C_p of solid	1.94 (kJ/kg-K)

Boiling temperature	100°C	Enthalpy of vaporization	2257 kJ/kg
Solidification temperature	0°C	Enthalpy of fusion	333.4 kJ/kg

Note: Enthalpy of vaporization is the energy required to turn a saturated liquid into a saturated vapor, and enthalpy of fusion is the energy required to turn a solid into a saturated liquid.

Suppose you mix the following in a constant pressure vessel and allow sufficient temperature for the entire mixture to reach an equilibrium temperature:

1 kg of vapor at 200°C
1 kg of solid at -40°C

Determine the final temperature of the mixture, and if applicable, determine the quality.

$Q - \cancel{W}^{0} = \Delta U + \cancel{\Delta KE + \Delta PE}$

$Q = \Delta U = m C_p \Delta T$

For solid

Heating to melting temp: $Q_1 = (1\text{ kg})(1.94 \frac{kJ}{kg\,°C})(40°C) = 77.6\text{ kJ}$

Melting completely: $Q_2 = (1\text{ kg})(333.4\text{ kJ/kg}) = 333.4\text{ kJ}$

Heating to final temperature: $Q_3 = (1\text{ kg})(4.19 \frac{kJ}{kg\,°C})\Delta T = 4.19 T_f$ if $T_f = 100°C$ $Q_3 = 419\text{ kJ}$

For vapor

Cooling to boiling temp: $Q_4 = (1\text{ kg})(1.17 \frac{kJ}{kg\,°C})(100°C) = 117\text{ kJ}$

Completely condensing: $Q_5 = (1\text{ kg})(2257. \frac{kJ}{kg}) = 2257\text{ kJ}$ ← Big

Since the heat release from condensing the vapor is large, all of the ice will be melted

Heat required to get ice to boiling temperature: $Q_{ice} = Q_1 + Q_2 + Q_3 = 830\text{ kJ}$

After vapor reaches boiling temp: $Q_{rem} = 830 - 117\text{ kJ} = 713\text{ kJ}$

↑ this causes some vapor to condense.

$m_{cond} = \dfrac{713\text{ kJ}}{2257\text{ kJ/kg}} = 0.316\text{ kg}$ $\quad m_{vap} = 1 - 0.316\text{ kg} = 0.684\text{ kg}$

$X = \dfrac{m_{vap}}{m_{total}} = \dfrac{0.684}{2} \rightarrow \boxed{X = 0.342}$ $\quad \boxed{T_f = 100°C}$

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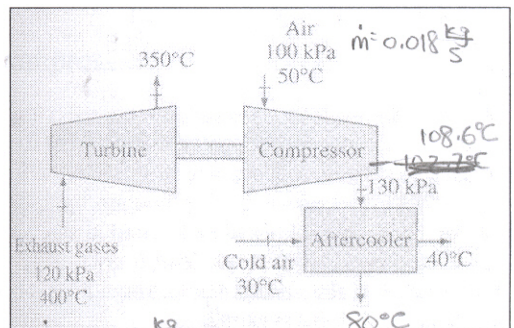
Question 2: [20 points]

$R_{air} = 0.287 \frac{kPa \cdot m^3}{kg \cdot K}$

In a turbocharger, exhaust gas enters the turbine at 400ºC and 120 kPa at a rate of 0.02 kg/s and leaves at 350ºC. Air enters the compressor at 50ºC and 100 kPa and leaves at 130 kPa at a rate of 0.018 kg/s. The compressor increases the air pressure with a side effect: It also increases the air temperature, which increases the possibility of a gasoline engine to experience engine knock. To avoid this, an aftercooler is placed after the compressor to cool the warm compressed air by cold ambient air before the compressed air enters the engine cylinders. It is estimated that the aftercooler must decrease the compressed air temperature below 80ºC to avoid knock. The cold ambient air enters the aftercooler at 30ºC and leaves at 40ºC. Disregard any frictional losses in the turbine and compressor and treating the exhaust gas as air, determine:

a) the temperature of the COMPRESSED air at the compressor outlet
b) the minimum volume flow rate of ambient air required to avoid knock.

State any assumptions that are made in your solution.
$C_{p,air\ @400ºC} \approx C_{p,air\ @350ºC} = 1.063$ kJ/kg·K
$C_{p,air\ @50ºC} = 1.008$ kJ/kg·K
$C_{p,air\ @30ºC} = 1.005$ kJ/kg·K



$\dot{m} = 0.02 \frac{kg}{s}$

[5]
Assume:
1) Steady-flo[w]
2) Air is id[eal] gas
3) Comp & turb are adiabatic
4) Comp/turb are 100% efficient
5) $\Delta pe \approx \Delta ke \approx 0$

[5] For turbine: $\cancel{\dot{Q}_{in}} + \cancel{\dot{W}_{in}} + \dot{m}_{in}(h_1 + \cancel{pe_1} + \cancel{ke_1}) = \cancel{\dot{Q}_{out}} + \dot{W}_{out} + \dot{m}(h_2 + \cancel{pe_2} + \cancel{ke_2})$

$W_{out} = \dot{m}(h_1 - h_2)$

for ideal gas: $\Delta h = C_p \Delta T$

$\dot{W}_{out} = \dot{m} C_p (T_1 - T_2) = (0.02 \frac{kg}{s})(1.063 \frac{kJ}{kg \cdot ºC})(400 - 350ºC)$

$\dot{W}_{out} = 1.063$ kW

[5] For compressor: $\cancel{\dot{Q}_{in}} + \dot{W}_{in} + \dot{m}_{in}(h_1 + \cancel{pe_1} + \cancel{ke_1}) = \cancel{\dot{Q}_{out}} + \cancel{\dot{W}_{out}} + \dot{m}_{out}(h_2 + \cancel{pe_2} + \cancel{ke_2})$

$\dot{W}_{in} = \dot{m}(h_2 - h_1) = \dot{m} C_p (T_2 - T_1)$

$T_2 = \frac{\dot{W}_{in}}{\dot{m} C_p} + T_1 = \frac{1.063\ kJ/s}{(0.018 \frac{kg}{s})(1.008 \frac{kJ}{kg \cdot ºC})} + 50ºC \rightarrow \boxed{T_2 = 108.6\ ºC}$

[5] For After cooler:

$(0.018 \frac{kg}{s})(\cancel{1.008 \frac{kJ}{kg \cdot K}})(108.6 - 80) = \dot{m}(\cancel{1.008 \frac{kJ}{kg \cdot K}})(40 - 30)$

$\dot{m} = 0.05148$ kg/s

$Pv = RT \rightarrow v = \frac{RT}{P} = \frac{(0.287 \frac{kPa \cdot m^3}{kg \cdot K})(30 + 273 K)}{100\ kPa} = 0.8[68]$

$\dot{V} = \dot{m} v = (0.05148 \frac{kg}{s})(0.868 \frac{m^3}{kg}) =$

$\rightarrow \boxed{\dot{V} = 0.0447 \frac{m^3}{s}}$

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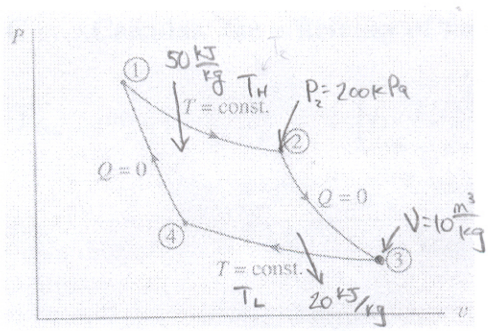
Question 3: [10 points]

A Carnot engine operating on **air** accepts 50 kJ/kg of heat and rejects 20 kJ/kg. Calculate the high and low reservoir temperatures if the maximum specific volume is 10 m^3/kg and the pressure after isothermal expansion is 200 kPa.

Hint: isentropic (adiabatic & reversible) compression and expansion processes satisfy the relationship Pv^k = const. For air, k=1.4, and R_{air} = 0.287 kJ/kg·K.



P-v diagram for Carnot engine

1→2: Isothermal expansion

2→3: Adiabatic & reversible expansion

3→4: Isothermal compression

4→1: Adiabatic & reversible compression

[2] $\eta_{th} = \cancel{1-}\frac{Q_L}{Q_H} = \cancel{1-}\frac{T_L}{T_H} \rightarrow \frac{20}{50} = \frac{T_L}{T_H} = 0.4 \Rightarrow \frac{T_3}{T_2} = 0.4$

Considering process 2→3: $P_1 V_1^k = P_2 V_2^k$ $\quad Pv = RT \rightarrow v = \frac{RT}{P}$

$P_1^{1-k}(\cancel{R}T_1)^k = P_2^{1-k}(\cancel{R}T_2)^k$

[5] $\left(\frac{T_1}{T_2}\right)^k = \left(\frac{P_2}{P_1}\right)^{1-k} \rightarrow \frac{T_1}{T_2} = \left(\frac{P_2}{P_1}\right)^{\frac{1-k}{k}} \rightarrow \left(\frac{T_3}{T_2}\right) = \left(\frac{P_2}{P_3}\right)^{\frac{1-k}{k}} = 0.4$ $\quad$ (P_2 = 200 kPa)

$\frac{P_2}{P_3} = (0.4)^{\frac{1.4}{1-1.4}} \rightarrow P_3 = \frac{200\text{ kPa}}{(0.4)^{\frac{1.4}{1-1.4}}} = 8.1\text{ kPa}$

[2] For state 3: $T_3 = \frac{P_3 V_3}{R} = \frac{(8.1\text{ kPa})(10\ m^3/kg)}{0.287\ \frac{kJ}{kg \cdot K}} = 282\text{ K}$ $\quad \boxed{T_L = 282\text{ K}}$

[1] $T_H = \frac{T_L}{0.4} \rightarrow \boxed{T_H = 705.2\text{ K}}$

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Question 4: [10 points]

A Carnot engine delivers 100 kW of power by operating between temperature reservoirs at 100°C and 1000°C.

A……Calculate the entropy change of each reservoir after 20 minutes.

B……Calculate the net entropy change of the two reservoirs after 20 minutes of operation.

C……Calculate the efficiency of the engine

[2] $\eta_{th,rev} = 1 - \frac{373}{1273} = \boxed{0.707}$ $\dot{W}_{out} = 100\,kW$

$= \frac{\text{desired output}}{\text{req'd input}} = \frac{\dot{W}_{out}}{\dot{Q}_H} \Rightarrow \dot{Q}_H = 141.4\,kW$

$\dot{W}_{out} = \dot{Q}_H - \dot{Q}_L \rightarrow \dot{Q}_L = \dot{Q}_H - \dot{W}_{out} = 141.4 - 100\,kW = 41.4\,kW$

[3] $\Delta \dot{S}_{gen,H} = -\frac{\dot{Q}}{T} = \frac{-141.4\,\frac{kJ}{s}}{1273\,K} = -0.111\,\frac{kJ}{K \cdot s}$

$\Delta S_H = \left(-0.111\,\frac{kJ}{K \cdot s}\right)\left(20\,min \times \frac{60\,s}{min}\right)$

$\boxed{\Delta S_H = -133.3\,\frac{kJ}{K}}$

[3] $\Delta S_{gen,L} = \frac{\dot{Q}_L}{T_L} = \frac{41.4\,kJ/s}{373\,K} = 0.111\,\frac{kJ}{K \cdot s}$

$\boxed{\Delta S_L = 133.3\,\frac{kJ}{K}}$

[2] $\boxed{\Delta S_{tot} = \Delta S_H + \Delta S_L = 0}$

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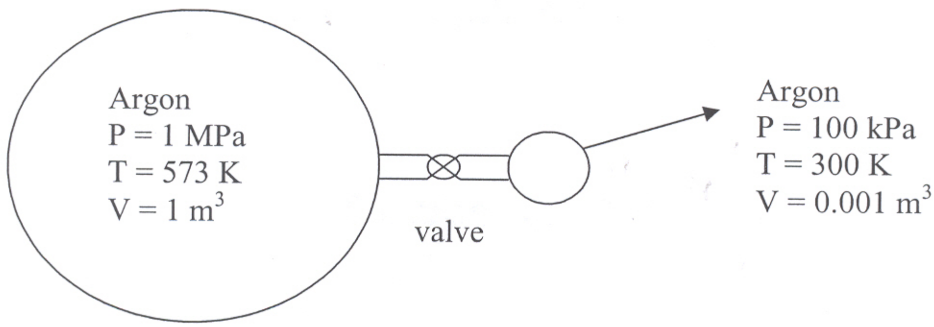
Question 5: [10 points]

Dibble wants to refill his Argon tank. The initial situation is depicted in the two tank system below. Assume no heat transfer is lost to the surroundings and there is no heat transfer from one tank to the other. Only mass flow is allowed through the valve. Determine if Dibble scorches his palms on the Argon tank or if he gets frostbite (ie. what is the final temperature?).

Cp Argon = 0.5203 kJ/kg*K
R_u = 8.314 kJ/kmol*K
M Argon = 39.948 kg/kmol

$R_{Ar} = \frac{8.314 \frac{kJ}{kmol\cdot K}}{39.948 \frac{kg}{kmol}} = 0.208 \frac{kJ}{kg\cdot K}$

$C_p = R + C_v \rightarrow C_v = 0.3122 \frac{kJ}{kg\cdot K}$



Note: not drawn to scale

Tank 1: $m_1 = \frac{PV}{RT} = \frac{(1000\ kPa)(1\ m^3)}{(0.208 \frac{kPa\cdot m^3}{kg\cdot K})(573\ K)} = 8.4\ kg$

Tank 2: $m_2 = \frac{PV}{RT} = \frac{(100\ kPa)(0.001\ m^3)}{(0.208 \frac{kPa\cdot m^3}{kg\cdot K})(300\ K)} = 0.0016\ kg$

Final pressures must be equal:

$P_{1f} = P_{2f} \rightarrow \frac{(m_{1i} - \Delta m) R T_{1f}}{V_1} = \frac{(m_{2i} + \Delta m) R T_{2f}}{V_2}$

Since the mass transfer out of the large tank is small, $m_{1i} - \Delta m \approx m_{1i} = 8.4\ kg$

$T_{1f} \approx T_{1i} = 573\ K$

$\frac{m_{1i} \cancel{R} T_{1i}}{V_1} = \frac{(m_{2i} + \Delta m) \cancel{R} T_{2f}}{V_2} \rightarrow \frac{V_2}{V_1} m_{1i} T_{1i} = (m_{2i} + \Delta m) T_{2f}$

① $4.8132 = 0.0016 T_{2f} + \Delta m T_{2f} = T_{2f}(0.0016 + \Delta m) \rightarrow T_{2f} = \frac{4.8132}{0.0016 + \Delta m}$

Energy balance for tank 2:

$(m_{2i} + \Delta m) u_{2f} = m_{2i} u_{2i} + \Delta m u_{1i}$

Compare to reference state of 0K

$(m_{2i} + \Delta m) C_v (T_{2f} - 0K) = m_{2i} C_v (T_{2i} - 0K) + \Delta m C_v (T_{1i} - 0K)$

$(0.0016\ kg + \Delta m)\cancel{(0.3122 \frac{kJ}{kg\cdot K})} T_{2f} = (0.0016\ kg)\cancel{(0.3122 \frac{kJ}{kg\cdot K})}(300\ K) + \Delta m \cancel{(0.3122 \frac{kJ}{kg\cdot K})}(573\ K)$

② $0.0016 T_{2f} + \Delta m T_{2f} = 0.48 + 573 \Delta m$

① into ②: $\frac{0.0077}{0.0016 + \Delta m} + \frac{4.8132 \Delta m}{0.0016 + \Delta m} = 0.48 + 573 \Delta m$

$0.0077 + 4.8132 \Delta m = 0.48(0.0016 + \Delta m) + 573 \Delta m (0.0016 + \Delta m)$

$0.0077 + 4.8132 \Delta m = (7.68 \times 10^{-4}) + 0.48 \Delta m + 0.9168 \Delta m + 573 \Delta m^2$

$0 = 573 \Delta m^2 - 3.4164 \Delta m - 0.006932$ $\quad \Delta m = 0.00756\ kg$ into ①

$\boxed{T_{2f} = 525.5\ K}$

$\boxed{T_{1f} \approx 573\ K}$