1	2	3	4	5	6	7	8	9	10	11	12	T
10	10	15	15	15	15	15	15	15	15	15	20	175

Math 1B, Final Examination

Section 1, 11-12am, N.Reshetikhin, May 19, 2003

Student's Name:

TA's name:

Student's i.d. number:

1. *10 pnts* Evaluate the integral

$$\int_0^1 x^3\sqrt{1+x^2}dx$$

2. *10 pnts* Evaluate the integral

$$\int x^2 \ln(1+x)dx$$

3. *15 pnts* Indicate which of the following statements are true and which are false. Do not show your work.

1. $\int_1^\infty \frac{\sin^2 x}{x^2} dx$ converges by comparison test with $\int_1^\infty \frac{1}{x^2} dx$.

2. $\int_1^\infty \frac{\sin^2 x}{x} dx$ diverges by comparison test with $\int_1^\infty \frac{1}{x} dx$.

3. $\int_0^2 \frac{dx}{(x-1)^2} dx$ is a convergent improper integral.

4. $\int_0^\infty \frac{1}{x^2} dx$ is a divergent improper integral.

5. $\int_0^\infty \frac{e^{-x}}{\sqrt{x}} dx$ is a convergent improper integral.

4. *15 pnts* Find the radius and the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \ln(n)\ x^n$$

5. *15 pnts* State whether the following series is absolutely convergent, conditionally convergent, or divergent. Do not show your work.

1. $\sum_{n=1}^{\infty}(-1)^n$.

2. $\sum_{n=1}^{\infty}\cos(\pi n)\frac{1+7n^2}{3n+14n^3}$.

3. $\sum_{n=1}^{\infty}(\sqrt{n+1}-\sqrt{n})$.

4. $\sum_{n=1}^{\infty}\frac{ne^n}{n!}(-1)^n$.

5. $\sum_{n=1}^{\infty}\cos(\pi n)\frac{1}{n\ln(n)}$

6. *15 pnts* For each statement indicate whether it is true or false. Do not show your work.

1. If $\sum_{n=1}^{\infty} c_n$ converges, then $\sum_{n=1}^{\infty} (-1)^n c_n$ also converges.

2. If $f(x) > 0$ is monotonically decreasing and $\int_{1000}^{\infty} f(x)dx < \infty$ then $\sum_{n=1}^{\infty} f(n)$ converges.

3. If the sequence $\{a_n\}$ converges and the sequence $\{b_n\}$ diverges then $\{a_n + b_n\}$ diverges.

4. If the sequence $\{a_n\}$ converges and and the sequence $\{b_n\}$ diverges then $\{a_n b_n\}$ diverges.

5. If $\sum_{n\geq 0} a_n 5^n$ converges and $\sum_{n\geq 0} a_n(-6)^n$ diverges, then $\sum_{n\geq 0} a_n 8^n$ diverges.

7. *15 pnts* Indicate whether each of the following series converges absolutely, converges conditionally, or diverges. Do not show your work.

1. $\sum_{n=1}^{\infty} \frac{n}{(n+1)^3}$

2. $\sum_{n=1}^{\infty} \frac{n^2}{(n+1)^3}$

3. $\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{(n+1)^3}$

4. $\sum_{n=1}^{\infty} (-1)^n \sin(\frac{1}{n}) \frac{n^2}{(n+1)^3}$

5. $\sum_{n=1}^{\infty} (-1)^n \sin(\frac{1}{n}) \frac{n^4}{(n+1)^3}$

8. *15 pnts* Find the general solution to the differential equation

$$xy' - y = x \ .$$

9. *15 pnts* Solve the initial-value problem

$$\frac{dy}{dx} = 1 + x^2 + y + x^2 y, \quad y(0) = 0 \ .$$

10. *15 pnts* Find the solution to the initial-value problem

$$y'' - y = e^x, \quad y(0) = 0, \quad y'(0) = 1 \ .$$

11. *15 pnts* Find the general solution to the differential equation:

$$y'' - 2y' - 3y = x \ .$$

12. *20 pnts* Find the power series solution to the differential equation:

$$y'' - xy = 0, \quad y(0) = 0, \quad y'(0) = 1 \ .$$