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155 Dwinelle Hall

Fall 1999, Math 1B
Final Examination

16 December, 1999
8:00-11:00 AM

1. (40 points, 4 points apiece) Find the following. If an expression is undefined, say so.

(a) $\int \frac{1}{x^4+x^3}\,dx$

(b) $\int_2^{\infty} x^{-3} e^{x^{-1}}\,dx$

(c) $\int_{-2}^{2} x^{-3} e^{x^{-1}}\,dx$

(d) $\lim_{n\to\infty} (0.999)^n$.

(e) The set of all real numbers p such that $\Sigma_{n=1}^{\infty} (-1)^n/(1+n^p)$ converges.

(f) The Maclaurin series (power series centered at 0) for $x\cos x$.

(g) The Maclaurin series for $\int_0^x e^{t^2}\,dt$ (as a function of x).

(h) $(1+i)^{-6}$

(i) The general solution to the differential equation $y''+4y'+4y = x^2$.

(j) The solution to the differential equation $y'' + y' + y = 1$ satisfying the initial conditions $y(0) = y'(0) = 1$.

2. (24 points, 8 points apiece) Find the following.

(a) The set of real numbers x such that the power series $\Sigma_{n=0}^{\infty} (n^5/8^n)(x-5)^n$ converges.

(b) The general solution to the differential equation $y' - y\tan x = e^{5x}$.

(c) The Maclaurin series for the solution to the differential equation $y'' - xy' - 2y = 0$ satisfying $y(0) = 0$, $y'(0) = 1$.

3. (12 points) For all $a\in(-\infty,\infty)$ and $x\in[0,\pi/2)$, find $\int_0^x (\cos t)^a (\sin t)^3\,dt$. You will find one formula that is valid for all x as above and *almost* all values of the exponent a, and other formulas for one or two special values of a. For full credit, you should obtain formulas covering all values of a, and specify the values of a for which each is valid, showing your work.

4. (12 points) Suppose y_1 is a solution to a second-order linear differential equation $y'' + P(x)y' + Q(x)y = G(x)$, and y_2 is a solution to an equation $y'' + P(x)y' + Q(x)y = H(x)$ with the same left-hand side, but in general a different right-hand side. Prove that y_1+y_2 is a solution to the equation $y'' + P(x)y' + Q(x)y = G(x) + H(x)$.

5. (12 points) Find the general solution of the differential equation $y'' + y = 1/x$ $(x>0)$. The solution will involve integrals that cannot be evaluated in terms of elementary functions, so in your answer, simply show those integrals, but do not attempt to evaluate them.