Math H1A Fall, 1995

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Midterm Exam—September 28, 1995

This is an "opcn book" cxam. You may consult your notes and textbook. Grading is based on completeness, clarity, and accuracy. Please write in complete English sentences and explain your reasoning carefully.

1 *(4 points)*. Find the domain of $g(x) = \sqrt{x^4 - 3x^2 - 4}$.

2 *(6 points)*. Use mathematical induction to prove that

$$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

for all $n \geq 1$.

3 *(5 points)*. Show that the axioms P10–12 on page 9 on Spivak imply that a^2 is positive for all non-zero real numbers a. [The first axiom states, for each real number a, that exactly one of the following holds: (i) $a = 0$; (ii) $a > 0$; (iii) $a < 0$. The other two state that the sum or product of two positive numbers is positive.] Using the axioms, prove that -1 is *not* the square of a real number.

4 *(8 points)*. Give a detailed proof from first principles (i.e., starting from the definition of "limit") that $x^2 \to 9$ as $x \to 3$.

5 *(5 points)*. For each real number t, define $f_t(x)$ by the recipe

$$f_t(x) = \begin{cases} -x+2 & \text{if } x > t \\ 3x-4 & \text{if } x \leq t. \end{cases}$$

For which t does $\lim_{x\to t} f_t(x)$ exist? Explain your reasoning without giving a formal proof.

6 *(7 points)*. Exhibit a function $f(x)$ defined on all of $\mathbf{R}$ with the following property: For each $M > 0$ there is a $\delta > 0$ such that $f(x) \geq M$ for all x with $0 < |x| < \delta$. Show that this property implies that $\lim_{x\to 0} f(x)$ does not exist.
