Name_____

TA & section_____

Math 1A — Final Exam

V.Jones, Fall 1999

450 points total. The first 10 questions are Multiple Choice, worth 10 points each. For each question mark an × in the most correct place in the grid below.
No partial credit for 1–10.
Questions 16 through 21 are not multiple choice.

	a	b	c	d	e
1					
2					
3					
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11					
12					
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14					
15					

GSI's only

MC_____

16_____

17_____

18_____

19_____

20_____

21_____

TOTAL_____

1. Which of the following is **most correct** concerning the function $y = f(x)$, which is differentiable at a.

   a) $f'(a) = \lim_{\delta \to 1} \dfrac{f(a+\delta) - f(a)}{\delta}$

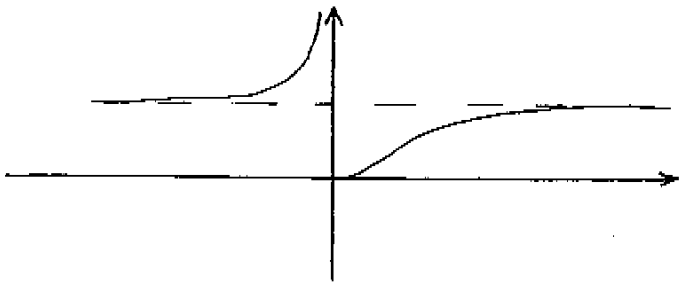
   b) $f'(a) = \lim_{\delta \to 0} \dfrac{f(a+\frac{1}{\delta}) - f(a)}{\delta}$

   c) $f'(a) = \lim_{\delta \to 0} \dfrac{f(a+\delta) - f(a)}{(\frac{1}{\delta})}$

   d) $f'(a) = \lim_{\delta \to a} \dfrac{f(a+\delta) - f(a)}{\delta}$

   e) $f'(a)$ is the limiting slope of the straight line obtained by zooming in more and more on the graph of $y = f(x)$ at $x = a$.

2. Which of the following functions (for $x \neq 0$) could have the following curve as its graph?



   a) $y = x \ln |x|$

   b) $y = e^{-\frac{1}{x}}$

   c) $y = xe^{x}$

   d) $y = xe^{-x}$

   e) $y = -xe^{-x}$

3. Newton's method used to solve the equation $\frac{1}{x} - a = 0$ yields the following sequence of approximations

   a) $x_{n+1} = 3x_n - ax_n^2$

   b) $x_{n+1} = 2x_n - ax_n^2$

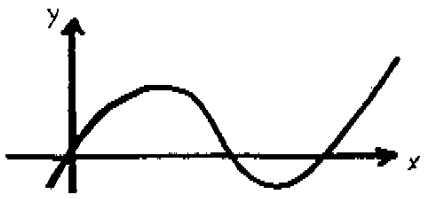
   c) $x_{n+1} = \frac{1}{x_n} - a$

   d) $x_{n+1} = x_n - \frac{1}{a}$

   e) $x_{n+1} = ax_n$

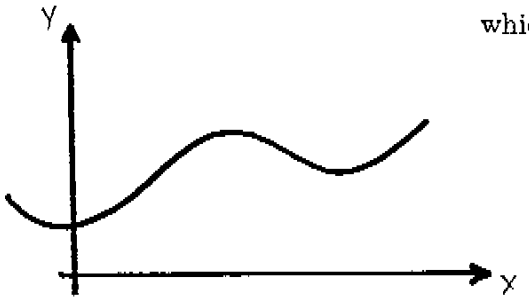
1

4. If the graph of f' is

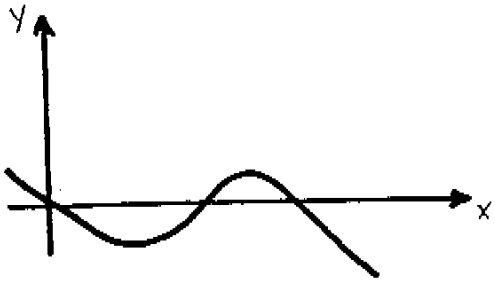


which of the following could be the graph of f?

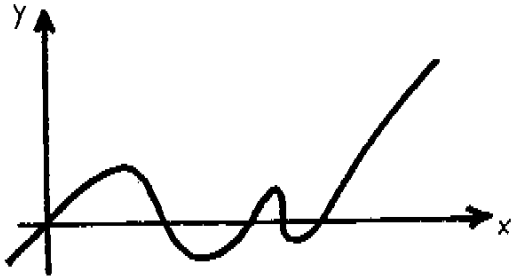
a)



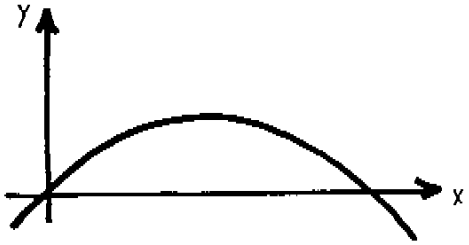
b)



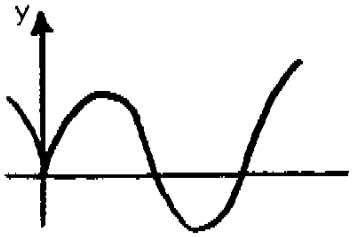
c)



d)



e)



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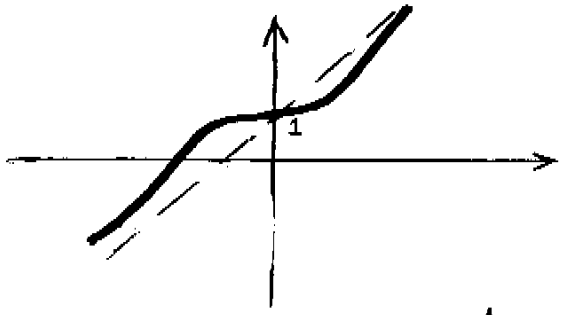
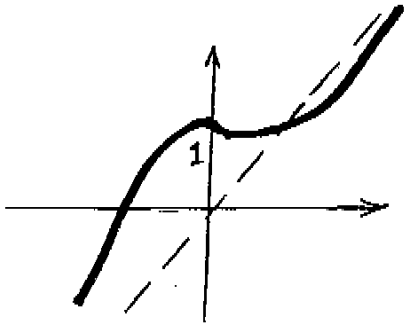
5. Which of the following best represents the graph of the function $f(x) = \dfrac{x^3+1}{x^2+1}$?

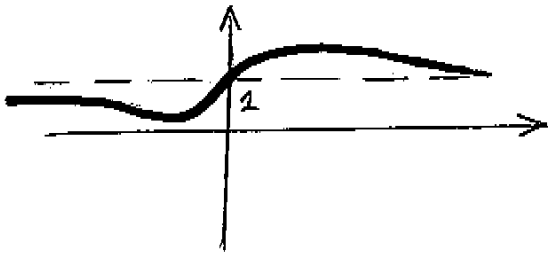
a)



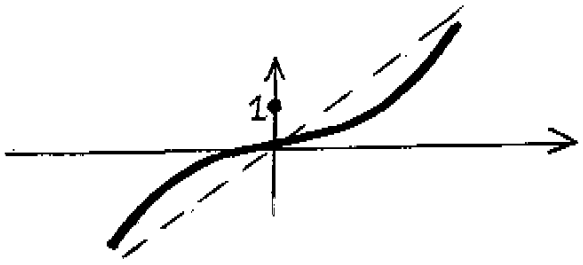
b)



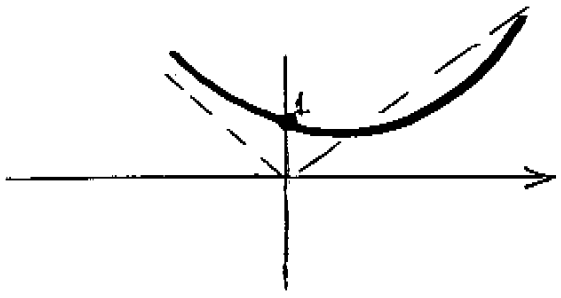
c)



d)



e)



3

6. Let f and g be two functions differentiable for all x and suppose that $f(a) = g(a)$ and $f''(x) > g''(x) + \frac{1}{2}$ for all $x > a$. Which of the following can we conclude?

   a) $f(x) > g(x)$ for all $x > a$

   b) $f(x) \geq g(x)$ for all $x > a$

   c) $f(x) > g(x)$ for all $x > c$ for some sufficiently large c

   d) $f'(x) > g'(x)$ for all $x > a$

   e) $f'(x) \geq g'(x)$ for all $x > a$

7. Which of the following functions is continuous?

   a) $f(x) = \begin{cases} \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$

   b) $f(x) = \begin{cases} \frac{1}{x} \sin \frac{1}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$

   c) $f(x) = \begin{cases} \sin \frac{1}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$

   d) $f(x) = \begin{cases} x \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$

   e) $f(x) = \begin{cases} x \sin \frac{1}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$

8. A ladder 10 ft long is leaning against a vertical wall. It starts to slide. When the point of contact of the ladder with the wall is 6 ft from the base of the wall that point of contact is moving down at 2 ft/sec. How fast is the point where the ladder touches the ground moving at that time?

   a) $1\frac{1}{2}$ ft/sec

   b) 2 ft/sec

   c) $2\frac{1}{3}$ ft/sec

   d) $2\frac{1}{2}$ ft/sec

   e) $2\frac{2}{3}$ ft/sec

4

9. The maximum value of $y = x^3 - 6x^2 + 9x$ on the interval [–2,0] is

a) 0

b) 4

c) 14

d) 16

e) It doesn't attain its maximum.

10. Which of the following best describes a solid whose volume is given by the formula $\int_0^2 2\pi y(4-y^2)dy$?

a) The solid obtained by rotating the region in the first quadrant bounded by $x = 4-y^2$ and the y-axis about the x axis.

b) The solid obtained by rotating the region in the first quadrant bounded by $x = y(4-y^2)$ and the y-axis about the x axis.

c) The solid obtained by rotating the region in the first quadrant bounded by $x = y(4-y^2)$ and the y-axis about the y axis.

d) The solid obtained by rotating the region in the first quadrant bounded by $x = 4-y^2$ and the y-axis about the y axis.

e) The solid obtained by rotating the region in the first quadrant bounded by $x = \sqrt{2y(4-y^2)}$ and the y-axis about the x axis.

11. $\lim_{x\to 0^+} \frac{1}{x^3} \int_0^x \sin(t^2)dt$ is

a) 0

b) 1

c) $\frac{1}{3}$

d) 3

e) –3

5

12. A point on the curve $y^2 = x^2 + 4x + 7$ closest to the origin is

a) $(-1, 2)$

b) $(1, \sqrt{12})$

c) $(0, \sqrt{7})$

d) $(2, \sqrt{19})$

e) $(-2, \sqrt{3})$

13. Which of the following functions is differentiable at $x = 0$?

a) $f(x) = \sqrt{1 + |x|}$

b) $f(x) = |x|$

c) $f(x) = \begin{cases} x^2 \sin(\frac{1}{x}) & x \neq 0 \\ 0 & x = 0 \end{cases}$

d) $f(x) = \begin{cases} \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$

e) $f(x) = \begin{cases} \sin x & x \geq 0 \\ \cos x & x < 0 \end{cases}$

14. If $f(x) = x \ln x$ then $f^{(10)}(x)$ is

a) $\frac{1}{10!}\, x^{10}$

b) $10! \ln(x)$

c) $\frac{1}{8!} x^8$

d) $8! \ln(x)$

e) $8! x^{-9}$

15. The area between the curves $y = 2x - x^2$ and $y = x^2$ for $0 \leq x \leq 2$ is

a) $-\frac{2}{3}$

b) $-\frac{4}{3}$

c) 0

d) 2

e) $\frac{4}{3}$

6

Longer Questions

16. *(50 pts)* Evaluate the following definite integrals.

(i) $\int \frac{\sin 2x}{\sin x}\, dx$

(ii) $\int \sqrt[3]{3-5x}\, dx$

(iii) $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}}\, dx$

(iv) $\int x\sqrt{x^2+a^2}\, dx$

(v) $\int \frac{\cos\left(\frac{\pi}{x}\right)}{x^2}\, dx$

17. *(60 pts)* Evaluate the following definite integrals.

(i) $\int_0^3 |6 - 9x + 3x^2| dx$

(ii) $\int_0^1 e^{\pi t} dt$

(iii) $\int_0^1 \frac{1}{x^2 + 1} dx$

(iv) $\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$

(v) $\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{x^2 \sin x}{1 + x^6} dx$

8

18. *(50 pts)* Evaluate the following limits.

(i) $\lim_{n\to\infty} \frac{1}{n}\left(\sum_{i=1}^{n} \frac{i}{n^8}\right)$

(ii) $\lim_{n\to\infty} \frac{1}{n}\left(\sum_{i=1}^{n} \sqrt{1-\frac{i^2}{n^2}}\right)$

(iii) $\lim_{x\to\infty} \frac{\ln x}{x^2}$

(iv) $\lim_{x\to 1^+} \frac{(x-1)}{\tan\left(\frac{\pi x}{2}\right)}$

(v) $\lim_{x\to 0} \frac{\sin x}{1+\cos x}$

19. *(50 pts)* Evaluate the following derivatives.

(i) $\frac{d}{dx}\left(\frac{x}{x^3+1}\right)$

(ii) $\frac{d^2}{dx^2}(e^{-x^2})$

(iii) $\frac{d}{dx}\int_x^{e^x} \frac{\cosh(t^2)}{1+t^4}\,dt$

(iv) $\frac{d}{dx}(x^{-x})$

(v) $\frac{d^{50}}{dx^{50}}(\sin x)$

10

20. *(45 pts)* The base of a solid is a square with vertices at (1,0), (0,1), (–1,0) and (0,–1). Each cross-section perpendicular to the x-axis is a semicircle. Find the volume of the solid.

Write answer here: _____

21. *(45 pts)* Find the volume of the solid obtained by rotating the bounded region between the curves $y = x^2$ and $y = x^3$ about the line $y = 2$.

Write answer here: _____

12