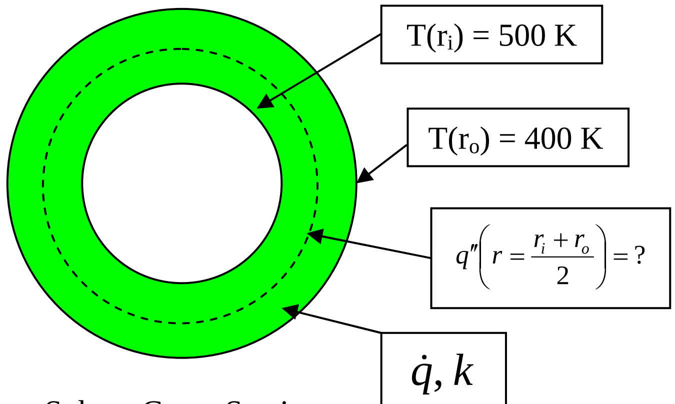
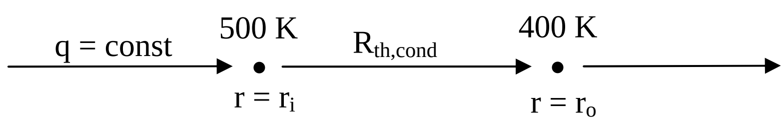
(30 Points)

1. Consider a spherical wall (or shell) that extends from $r = r_i$ to $r = r_o$. The inner surface temperature is 500 K and the outer surface temperature is 400 K. Determine the heat flux (units of W/m^2) at $r = (r_i + r_o)/2$.
(If you refer to an eq. or fig. in the text or notes give the page number)



Sphere Cross-Section

First find *q*, then find *q''*. Begin with resistive network:



For spherical geometries (equation 3.36 on page 122):

$$R_{th,cond} = \frac{1}{4\pi k}\left(\frac{1}{r_i} - \frac{1}{r_o}\right)$$

Heat transfer resistance equation:

$$\Delta T = qR \Rightarrow q = \frac{\Delta T}{R}$$

$\Delta T = 500 - 400 = 100\ K$, then

$$q = \frac{100}{4\pi k\left(\frac{1}{r_i} - \frac{1}{r_o}\right)} = \frac{400\pi k}{\left(\frac{1}{r_i} - \frac{1}{r_o}\right)}$$

Divide *q* by area of sphere at $r = (r_i + r_o)/2$ to get *q''*:

$$q'' = \frac{q}{A_{sph}} = \frac{q}{4\pi\left(\frac{r_i + r_o}{2}\right)^2} = \frac{400k}{\left(\frac{1}{r_i} - \frac{1}{r_o}\right)(r_i + r_o)^2}$$

Alternatively, Table 3.3 (pg. 126), under heat flux and spherical wall will produce the same result.

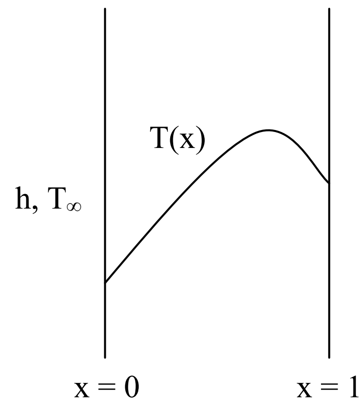
2. Consider an aluminum plate that extends from x = 0 to x = L = 1 m and contains a heat source. The surroundings are at 300 K. The temperature distribution is given by (units of K) $410 + 50x - 40x^2$. Determine the numerical value of the convection heat transfer coefficient at x = 0.
(If you refer to an equation or figure in the text or notes, give the page number)

$k_{Al} = 237$ @ T = 300 K {Table A.1, page 229}

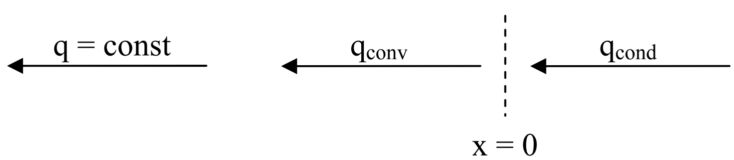
Given:

$T(x) = 410 + 50x - 40x^2$

$T_\infty = 300$ K



Use temperature distribution to find T(x = 0) = 410 K, thus heat transfer is to the left. At the left boundary the heat rate balance is:



At the left boundary (with q defined to the left), the heat transfer rates can be found as:

$$q''_{conv} = h\left(T_s - T_\infty\right)$$

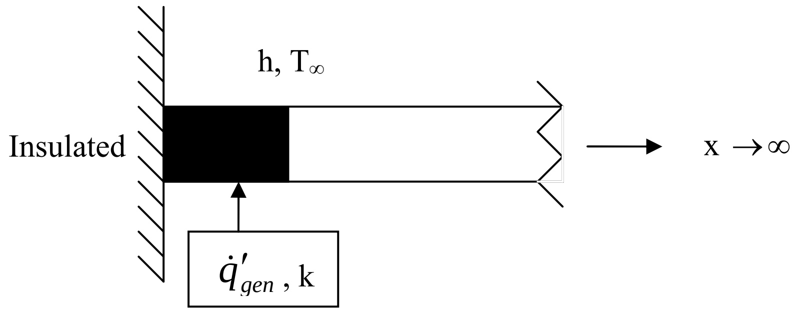
$$q''_{cond} = k\frac{dT}{dx}\Big|_{x=0}$$

Setting these two quantities equal and evaluating for *h* gives:

$$h = \frac{k\frac{dT}{dx}\Big|_{x=0}}{T_s - T_\infty} = \frac{237\,\frac{W}{mK}\,50\,\frac{K}{m}}{110K} = 107.7\,\frac{W}{m^2K}$$

(60)

3. An infinitely long fin contains a heat source from its base at x = 0 to x = 3 m. The heat source is a constant and generates 5 W/m. The base of the fin (at x = 0) is insulated. Determine (a) the heat loss from the fin to the surroundings (the surroundings are at 300 K) and (b) the temperature of the fin at x = 4 m.



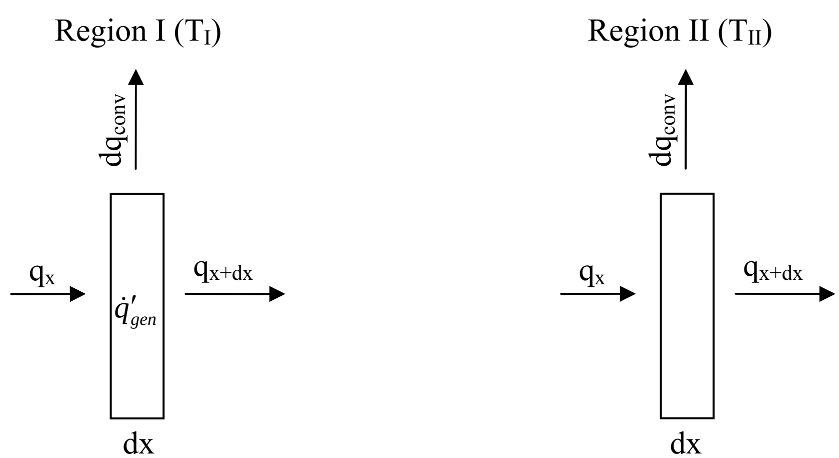
A)

At steady state, the cumulative heat loss must equal the heat generation rate. That is, the total heat loss, q_{loss} is:

$$q_{loss} = \dot{q}'_{gen}\,L = 5\tfrac{W}{m}\,3m = 15\ W$$

B)

This part requires the solution of the temperature profile in both regions. Set up a control volume for each of the two regions.



$q_x - q_{x+dx} - dq_{conv} + dq_{gen} = 0$ $\qquad$ $q_x - q_{x+dx} - dq_{conv} = 0$

$$-\frac{d}{dx}(q_x)dx - dq_{conv} + dq_{gen} = 0 \qquad -\frac{d}{dx}(q_x)dx - dq_{conv} = 0$$

$$\frac{d^2T_I}{dx^2} - \frac{hP}{kA}\left(T_I - T_\infty\right) + \frac{\dot{q}'_{gen}}{kA} = 0 \qquad\qquad \frac{d^2T_{II}}{dx^2} - \frac{hP}{kA}\left(T_{II} - T_\infty\right) = 0$$

Let $\theta_I = T_I - T_\infty - \frac{\dot{q}'_{gen}}{hP}$, $\frac{d\theta_I}{dx} = \frac{dT_I}{dx}$ Let $\theta_{II} = T_{II} - T_\infty$, $\frac{d\theta_{II}}{dx} = \frac{dT_{II}}{dx}$

Let $m^2 = \frac{hP}{kA}$, $m = \sqrt{\frac{hP}{kA}}$, where P is perimeter and A is area of cross section.

Set up temperature profile in both regions:

$$T_I = A\ \cosh\left(mx\right) + T_\infty + \frac{\dot{q}'_{gen}}{hP}$$

(drop sinh(mx) term because symmetric at x = 0 due to insulated BC)

$$T_{II} = B\exp\left(-mx\right) + T_\infty$$

(drop exp(mx) term because must be bounded as x tends toward ∞)

Apply temperature continuity at x = 3:

$$T_I\left(x=3\right) = T_{II}\left(x=3\right) \Rightarrow B = \exp\left(3m\right)\left[A\ \cosh\left(3m\right) + \frac{\dot{q}'_{gen}}{hP}\right]$$

Apply heat rate continuity at x = 3:

$$\frac{dT_I}{dx}\Big|_{x=3} = \frac{dT_{II}}{dx}\Big|_{x=3} \Rightarrow B = -A\ \sinh\left(3m\right)\exp\left(3m\right)$$

Combining these equations to solve for A gives:

$$A = \frac{-\dot{q}'_{gen}}{hP}\exp\left(-3m\right)$$

Substituting A back into solve B gives:

$$B = \frac{\dot{q}'_{gen}}{hP}\sinh\left(3m\right)$$

Recast the temperature solution in region II, T_{II}:

$$T_{II} = T_\infty + \frac{\dot{q}'_{gen}}{hP}\sinh\left(3m\right)\exp\left(-mx\right), \qquad \left\{x >= 3\right\}$$

Solve for T(x = 4) gives:

$$T\left(x=4\right) = T_\infty + \frac{\dot{q}'_{gen}}{hP}\sinh\left(3m\right)\exp\left(-4mx\right)$$