Math 1A Final 2004-12-15 12:30-3:30

You are allowed 1 sheet of notes. Calculators are not allowed. Each question is worth 1 mark, which will be given only for a clear correct answer and correct working. There is no partial credit for wrong answers.

1. Find a formula for the inverse of the function $y = 3x^3 + 2$.
2. Evaluate the limit $\lim_{h\to 0} \frac{(2+h)^3-8}{h}$.
3. Prove that $e^x = 2 + x$ has at least one real root.
4. Differentiate $f(x) = (ax + b)/(cx + d)$.
5. Find the derivative of the function $y = \tan^3(2x)$.
6. Find dy/dx if $4\cos(x)\sin(y) = 1$.
7. Find the critical numbers of the function $f(x) = xe^{2x}$.
8. Show that the equation $x^4 + 4x + c = 0$ has at most two real roots.
9. Find $\lim_{x\to 0} \frac{\cos(x)-1}{x^2}$.
10. Find the points on the ellipse $4x^2 + y^2 = 4$ that are farthest away from the point $(1, 0)$.
11. Explain why Newton's method does not work for finding the root of the equation $x^3 - 3x + 6 = 0$ if the initial approximation is chosen to be $x_1 = 1$.
12. Use one iteration of Newton's method applied to the initial approximation $x_1 = 2$ to find $36^{1/5}$ correct to two decimal places.
13. Find a function f such that $f'(x) = x^3$ and the line $x + y = 0$ is tangent to the graph of f.
14. Find f given that $f''(x) = x^{-2}, x > 0$, $f(1) = 0$, $f(2) = 0$.
15. Estimate the area under the graph of $f(x) = 1/x$ from $x = 1$ to $x = 4$ using three rectangles and right endpoints. Sketch the graph and rectangles.
16. If $\int_1^5 f(x)dx = 12$ and $\int_2^5 f(x)dx = 14$ find $\int_1^2 f(x)dx$.
17. Evaluate the integral $\int_{-2}^{2} \sqrt{4 - x^2}dx$ by interpreting it as an area.
18. Prove that $2 \le \int_{-1}^{1} \sqrt{1 + x^2}dx \le 2\sqrt{2}$.
19. Find the derivative of the function $g(x) = \int_1^x \ln(t)dt$.
20. Find the derivative of $y = \int_{\sin(x)}^{x} \cos(t)dt$.
21. Evaluate the integral $\int_{-1}^{0}(2x - e^x)dx$.
22. Evaluate the integral $\int_0^{\pi/4} \frac{1+\cos^2(\theta)}{\cos^2(\theta)}d\theta$.
23. Evaluate the indefinite integral $\int y^3\sqrt{2y^4 - 1}dy$.
24. Evaluate the indefinite integral $\int \tan(x)dx$.
25. Evaluate the definite integral $\int_0^4 (x - 2)^7 dx$.
26. By comparing areas, show that $1 + 1/2 + 1/3 + \cdots + 1/(n - 1) > \ln(n)$ if $n \ge 2$.
27. Find the area enclosed by the curves $4x + y^2 = 12$, $x = y$.
28. Find the volume of the region obtained by rotating the region bounded by the curves $y = e^x$, $y = 0$, $x = 0$, $x = 1$, about the x-axis.
29. Use the method of cylindrical shells to find the volume of a sphere of radius r.
30. Find the average value of $\sqrt{x}$ on $[0, 4]$.