Math 1A Final 2005-12-15 5:00-8:00pm R. Borcherds

You are allowed 1 sheet of notes. Calculators are not allowed. Each question is worth 3 marks, which will only be given for a clear and correct answer in simplified form.

1. Draw the graph of the function $y = |\cos(x)|$ for $-\pi \le x \le \pi$.
2. Evaluate the limit $\lim_{x\to 9} \frac{x^2-81}{\sqrt{x}-3}$.
3. Prove that $x^4 + 1 = 3x$ has at least one real root.
4. Differentiate $e^x/(x+1)$.
5. Find the derivative of the function $y = \cos(\cos(\cos(x)))$.
6. Find dy/dx if $x^2y + xy^2 = 2x$.
7. Find the derivative $D^{57}\sin(2x)$. (D means d/dx)
8. If $f(1) = 10$ and $f'(x) \ge -1$ for all x, what is the smallest possible value of $f(5)$?
9. Find $\lim_{x\to+\infty} x^{1/x}$.
10. Sketch the curve $y = x\ln(x)^2$ for $x > 0$.
11. Find two numbers whose difference is 10 and whose product is a minimum.
12. Use one iteration of Newton's method applied to the initial approximation $x_1 = 2$ to estimate $9^{1/3}$.
13. Find a function f such that $f'(x) = x^3$ and the line $x + y = 0$ is tangent to the graph of f.
14. Find f given that $f''(x) = \sin(x)$, $f(0) = 1$, $f'(0) = 0$.
15. Estimate the area under the graph of $f(x) = x^2$ from $x = 1$ to $x = 4$ using three rectangles and left endpoints. Sketch the graph and rectangles.
16. If $\int_1^5 f(x)dx = 12$ and $\int_1^4 f(x)dx = 14$ find $\int_4^5 f(x)dx$.
17. Evaluate the integral $\int_0^3 (1 + \sqrt{9-x^2})dx$ by interpreting it as an area.
18. Prove that $1/e \le \int_0^1 e^{-x^2}dx \le 1$.
19. Find the derivative of the function $g(x) = \int_0^x e^{-t^2}dt$.
20. Find the derivative of $y = \int_{\cos(x)}^{\sin(x)} \tan(t)dt$.
21. Evaluate the integral $\int_{-1}^1 (x^3 + 2x + 1)dx$.
22. Evaluate the integral $\int_0^{\pi/4} \sec(\theta)\tan(\theta)d\theta$.
23. Evaluate the indefinite integral $\int (1+y^2)^{10} y dy$.
24. Evaluate the indefinite integral $\int \tan(x)\ln(\cos(x))dx$.
25. Evaluate the definite integral $\int_1^e \frac{\ln(x)^3}{x}dx$.
26. By comparing areas, show that $1 + 1/2 + 1/3 + \cdots + 1/(n-1) > \ln(n)$ if $n \ge 2$.
27. Find the area enclosed by the curves $y = x^2$, $y = 2/(x^2+1)$.
28. Find the volume of the region obtained by rotating the region bounded by the curves $y = \sqrt{x-1}$, $y = 0$, $x = 2$, $x = 10$, about the x-axis.
29. Use the method of cylindrical shells to find the volume generated by rotating the region bounded by $y = x^2$, $y = 0$, $x = 1$ about the y-axis.
30. Find the average value of $\sin(x)^2$ on $[0, 2\pi]$.