Math 1A Professor K. Ribet Fall, 1993

First Midterm Exam — September 23, 1993
80 minutes

1a *(5 points)*. Express the derivative $\frac{d}{dx}\left\{x\sin(\frac{1}{x^2})\right\}\Big|_{x=23}$ as a limit. Do not evaluate the limit.

1b *(5 points)*. Use the definition of the derivative to calculate $f'(1)$ when $f(x)=\sqrt[3]{x}$. The formula $b^3-a^3=(b-a)(b^2+ab+a^2)$ may be helpful.

2. In the problems on this page, you may use the differentiation formulas we have derived in class:

a *(5 points)*. Find the equation of the line tangent to the curve $y=\frac{x^2}{x^3+1}$ at the point $(1,\frac{1}{2})$.

b *(5 points)*. A sugar cube tumbles from a 98-meter tall campanile. How fast is it falling after t seconds? [If you use the formula $s(t)=4.9t^2$, explain in a sentence or two what it means.]

c *(2 points)*. How fast is the cube falling when it hits ground?

3 *(8 points)*. What is the domain of the function $g(t)=\sqrt{\frac{(t-1)(t-5)}{(t-3)(t-5)}}$? Find the horizontal and vertical asymptotes of the curve $u=g(t)$. Make a crude sketch of this curve, showing the asymptotes.

4a *(5 points)*. Find $\lim_{t\to 5^-}\left(\frac{[t]}{[-t]}\cdot|-t|\right)$, where [ ] is the "greatest integer" function.

4b *(5 points)*. Find $\lim_{t\to-\infty}\left(\sqrt{t^2-t+2}-\sqrt{t^2+t+1}\right)$.

5 *(5 points each)*.

a. Evaluate $\lim_{b\to 2}\frac{b^{691}-2^{691}}{b-2}$ by using rules of differentiation, first expressing the limit as a derivative.

b. Suppose that $f(\pi/2)=12$, $f'(\pi/2)=3$. Using the methods discussed in class, calculate $\lim_{\theta\to\pi/2}\frac{\cos(\theta)}{f(\theta)-12}$.

6 *(5 points each)*.

a. Find $f'(x)$ if $f(x) = \frac{1}{x^2}\sin(x)$.

b. Find $\lim_{x\to\infty} \frac{\sin(x)}{x^2}$. Explain your reasoning.

Second Midterm — October 28, 1993
80 minutes

1a *(5 points)*. A sample of chalk contains 0.3 grams of radioactive dwinellium, which has a half-life of 18 years. How many years must expire before the sample contains only 0.01 grams of radioactive dwinellium?

1b *(4 points)*. Find all possible values of $\cosh x$, given that $\sinh x = 5/12$.

2a *(5 points)*. Use differentials to find an approximate value for $\sin 1^\circ$.

2b *(5 points)*. Let g be the function inverse to f. Calculate $g'(2)$ from the table

x	0	1	2	3	4
$f(x)$	2	4	1	3	5
$f'(x)$	$\frac{1}{2}$	-1	0.2	$-\frac{1}{3}$	9

3. Find the derivative $\frac{dy}{dx}$ *(each part is worth four points)*:

a. $y = \arcsin(\sqrt{x})$

b. $y = e^{x^3+1}$

c. $y = \log_x(\cos x)$ $(0 < x < \pi/2)$.

4 *(8 points)*. A slug and an ant left the base of the Campanile at midnight last night. The ant began moving directly north, toward Evans Hall. The slug moved east, toward a slugfest in Birge Hall. At 8AM this morning, the ant had traveled 60 feet and was moving north at 10 feet/hour. The slug was 80 feet east of the Campanile, but had started moving *west* at the rate of 5 feet/hour. At what rate was the distance between the slug and the ant changing at 8AM?

5 Evaluate the limits *(four points each)*:

a. $\lim_{x\to 0+} \frac{x}{x^2+125}$

b. $\lim_{n\to\infty} \left(1 + \frac{\ln 2}{n}\right)^n$

c. $\lim_{t\to 0^-}\left(\frac{1}{t}-\frac{1}{e^t-1}\right)$

6a *(4 points)*. Find $\frac{dy}{dx}$ if $y = x^y$.

6b *(3 points)*. Find the derivative y', given $y^2 + 6xy + x^2 = 7$.

6c *(2 points)*. Find a formula for y'' in terms of x, y and y' if $y^2 + 6xy + x^2 = 7$.

Final Exam — December 15, 1993

1a *(5 points)*. Differentiate with respect to x: $\sqrt{x+\sqrt[3]{x}}$.

1b *(5 points)*. Find $L(-4)$ given that $L(-1) = 1$ and that $L'(x) = \frac{1}{x}$.

2a *(6 points)*. Find $\lim_{x\to 0} f(x)^{g(x)}$, where $f(x) = (0.4)^{(x^{-2})}$ and $g(x) = 3x^2$.

2b *(5 points)*. Evaluate $\lim_{x\to 1+}\frac{\ln|x|}{|2-x-x^2|}$.

3a *(5 points)*. Evaluate $\int \frac{dx}{x[1+(\ln x)^2]}$.

3b *(6 points)*. Evaluate $\int_{\pi/3}^{\pi/2}\frac{\cos x}{\sin x}\,dx$, simplifying your answer as much as possible.

4 *(9 points)*. A rain gutter is to be constructed from a metal sheet of width 30cm by bending up one-third of the sheet on each side through an angle θ. How should θ be chosen so that the gutter will carry the maximum amount of water? (A crude hand-drawn diagram was supplied.)

5a *(6 points)*. Evaluate $\lim_{n\to\infty}\frac{3}{n}\sum_{k=1}^{n}\cos\left(\frac{n+3k}{n}\right)$.

5b *(5 points)*. Let c be a real number. Show that the equation $x^5 - 6x + c = 0$ has at most one root in the interval $[-1, 1]$.

6a *(6 points)*. Find all numbers a such that the line tangent to $y = x^2 + 1$ at the point $(a, a^2 + 1)$ passes through $(0, -8)$.

6b *(5 points)*. Find the derivative of $(\sin x)^{\tan x}$ with respect to x.

Suppose that $\mathcal{R}$ is the region lying between the graphs of $y = x^3$ and $y = 27x$ in the part of the plane where x and y are positive.

7a *(5 points)*. Find a definite integral which represents the area of $\mathcal{R}$.

7b *(6 points)*. Find a definite integral which represents the volume of the figure generated by revolving $\mathcal{R}$ about the y-axis.

8a *(6 points)*. Find $\frac{d}{dx}\int_{\sin x}^{\cos x}\sqrt{t^3+1}\,dt$.

8b *(4 points)*. Find the average value of $\sin x$ on the interval $[0,\pi]$.

9a *(5 points)*. Evaluate $\lim_{h\to 0}\frac{\sinh\left(\frac{\pi}{2}+h\right)-\sinh\left(\frac{\pi}{2}-h\right)}{h}$.

9b *(5 points)*. Find $\frac{dy}{dx}$ at the point $(3,1)$ on the curve $2(x^2+y^2)^2=25(x^2-y^2)$.

10a *(2 points)*. Bob and Jill lift a 90-pound Stairmaster over a distance of 30 feet. How much work do they perform?

10b *(4 points)*. At 7PM, a large pizza is taken from a 415°F oven to a 65°F dining room. At 7:08PM, the pizza has cooled to 135°F. What is the temperature of the piece which remains at 7:16PM? (Assume the validity of Newton's law of cooling— the pizza cools at a rate proportional to the difference of its temperature and that of the room.)