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2050 VLSB

Fall 1998, Math 1AW
**First Midterm Exam**

2 October, 1998
3:10-4:00 PM

**1.** (32 points, 8 points apiece) Compute the following limits. Give the value if the limit is defined, or if it is $\infty$ or $-\infty$. If none of these is true, write *No limit.*

(a) $\lim_{x \to \infty} \dfrac{5 + x^3}{5 - x^2}$.

(b) $\lim_{x \to 0} \dfrac{e^{3x} - 1}{x}$.

(c) $\lim_{x \to 2} \dfrac{x^2 - 4}{\sqrt{x} - \sqrt{4 - x}}$.

(d) $\lim_{x \to \infty} \sin x$.

**2.** (36 points, 9 points apiece) Compute the following derivatives. (Note that (c) is a second derivative.)

(a) $\dfrac{d}{dx}(x^5 + 2x^4 + 79)$

(b) $\dfrac{d}{dx}(\cos x^a)^b$, where $a$ and $b$ are real numbers.

(c) $\dfrac{d^2}{dx^2} e^{\sqrt{x}}$.

(d) $\dfrac{d}{dx} g(x)$, where $g$ is the inverse of the function $f(x) = x^3 + x$.

**3.** (12 points) A point $p$ is moving in the $(x, y)$-plane (happily unaware that three hundred and forty-four Math 1A students are thinking about it). At a certain moment, its position is $(x_0, y_0)$, its velocity in the $x$-direction is 1, and its velocity in the $y$-direction is 2. Find the *rate of change* of its distance from $(0, 0)$ (in terms of $x_0$ and $y_0$).

**4.** (a) (8 points) Suppose $f$ is a function and $a$ a real number such that $f$ is differentiable at $a$. Give the definition of the derivative $f'(a)$.

(b) (12 points) If $f$ and $g$ are functions, and $a$ a real number such that $f$ and $g$ are differentiable at $a$, prove from the above definition a formula for the derivative at $a$ of the function $H(x) = f(x) - g(x)$. (You may assume without proof results proved in Stewart about limits; but assume nothing about derivatives except the definition.)