George M. Bergman | Fall 1998, Math 1AW | 11 December, 1998
Pauley Ballroom | **Final Examination** | 12:30-3:30 PM

**1.** (50 points, 5 points apiece) Find the following. When asked for a limit, give a real value, $\infty$, or $-\infty$, or, if the function approaches none of these, the words *No limit.*

(a) $\lim_{x \to 1} \ 3^{x-2}$

(b) $\lim_{x \to \infty} \ \sqrt{x^2+x} - x$

(c) $\lim_{x \to -\infty} \ \sqrt{x^2+x} - x$

(d) $\frac{d}{dx} \ \frac{Ax+B}{Cx+D}$, where $A, B, C, D$ are constants.

(e) $\lim_{x \to 0} \ e^{(3^x - 4^x)/x}$

(f) $y'$, for $y$ a differentiable function satisfying $x^y = 2y^x$. The answer should be expressed in terms of $x$ and $y$. (Suggestion: use logarithmic differentiation.)

(g) $\frac{d^2}{dx^2} \ \sin^{-1} x$

(h) $\frac{d}{dx} \ \int_0^{x^2} e^{t^2} \, dt$

(i) $\int (\sin 2x + \sin 3x) \, dx$

(j) $\Sigma_{i=1}^{1000} \ (3i+5)$

**2.** (15 points) (a) (9 points) Suppose $f$ is a function such that $f'$ is defined and continuous on an open interval containing 0, and that $f(0) = 0$ but $f'(0) \neq 0$. Show that $\lim_{x \to 0} f(5x)/f(x) = 5$. You may use any facts given in the readings.

(b) (6 points) For $f(x) = x^2 + x^3$, determine $\lim_{x \to 0} f(5x)/f(x)$. If the answer is not 5, say briefly why the result of part (a) does not apply.

**3.** (22 points) (a) (8 points) A quick computation (which you should not do here) shows that $\frac{d}{dx} \ \ln \cos x = -\tan x$. With the help of this fact if needed, compute the area bounded by the curve $y = \tan x \ \ (0 \leq x \leq \pi/4)$, the $x$-axis, and the line $x = \pi/4$.

(b) (14 points) Let $R$ be the region bounded by the curve $y = \sin(\pi x^2)$ and the $x$-axis, between $x = 0$ and $x = 1$. Find the volume of the solid of revolution of $R$ about the line $x = 0$ (the $y$-axis).

**4.** (13 points) Let $f$ be a function defined on an interval containing the real number $a$. Complete the following sentences to give precise mathematical definitions. (For full credit on part (a), you should give the "$\epsilon$-$\delta$" definition. In each part, if you cannot give a mathematical definition, you may get partial credit for briefly giving the idea of the concept.)

(a) (5 points) If $L$ is a real number, we say that $\lim_{x \to a} f(x) = L$ if

(b) (4 points) The function $f$ is said to be continuous at $x = a$ if

(c) (4 points) The derivative of $f$ at $a$ (written $f'(a)$ or $\frac{d}{dx} \ f(x)|_{x=a}$) is defined as