P. Vojta
Fall 2001

Math 1A Final Examination

3 hours

1. (18 points) Find the following limits:
   (a). $\lim_{x\to 0}(e^x + x)^{1/x}$
   (b). $\lim_{x\to \frac{\pi}{2}^-} \frac{\tan^2 x - 3}{7\tan^2 x + \sin x}$
   (c). $\lim_{x\to -\infty} \frac{3x+2}{\sqrt{x^2+7}}$
2. (18 points) Find the following derivatives:
   (a). $\frac{d}{dx}\cosh^{-1} x$
   (b). $\frac{d}{dx}\frac{\ln(\cosh x^2)}{e^x+2}$
3. (18 points) (a). Find the slope of the curve $y^3 = x^4 + 8y - 9$ at the point $(1, 2)$.
   (b). Find y'' at $(1, 2)$.
4. (12 points) Use differentials to estimate $\tan\left(\frac{\pi}{4} + .05\right)$.
5. (12 points) (a). State fully and carefully the Mean Value Theorem.
   (b). Let $0 < a < b$. Show that there exists a number c such that $a < c < b$ and $c\ln\left(\frac{b}{a}\right) = b - a$.
6. (20 points) A cylindrical gob of goo is undergoing a transformation in which its height is decreasing at the rate of 1 cm/sec, while its volume is decreasing at the rate of 2π cm^3/sec. (It retains its cylindrical shape while all this is happening.) If, at a given instant, its volume is 24π cm^3 and its height is 6 cm, determine whether its radius is increasing or decreasing at that instant, and at what rate.
7. (15 points) Determine the maximum and minimum values of the function $f(x) = x - 3x^{2/3}$ on the interval $[-1, 27]$, and find all points (x-values) where they occur.
8. (12 points) Find all asymptotes (horizontal, vertical, and slant) of the function

$$f(x) = \frac{x^3 - 1}{x^2 - 1}.$$

9. (15 points) A cyclist traveling at 30 ft/sec decelerates at a constant 3 ft/sec^2. How many feet does he travel before coming to a complete stop?
10. (10 points) If Newton's method is used to solve $x^3+x^2+2 = 0$ with an initial approximation $x_1 = -2$, what is the second approximation, x_2?

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11. (35 points) Find the following integrals:

(a). $\int_{-1}^{1} \tan\left(\frac{\pi}{4}x^3\right)\, dx$

(b). $\int \frac{dx}{9x^2+1}$

(c). $\int \frac{x^4+1}{x^3}\, dx$

(d). $\int (x \sin x^2) e^{\cos x^2}\, dx$

(e). $\int_1^2 x\sqrt[4]{x-1}\, dx$

12. (20 points) Find the integral $\int_1^2 x^2\, dx$ using the definition as a limit of Riemann sums. You may use any (valid) choice of x_i^*, and may also use any of the formulas

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2 .$$

13. (20 points) (a). The region bounded by the curves $y = 2x^2$ and $y = 3x - x^2$ is rotated about the y-axis. Find the volume of the resulting solid.

(b). The same region is rotated about the line $y = -1$. Write an integral expressing the volume of the resulting solid. (DO NOT evaluate it.)