Name_____

TA & section_____

Math 1A — First Midterm

V.Jones, Fall 1999

B

200 points total. The first 12 questions are Multiple Choice. worth 10 points each. For each question mark an × in the most correct place in the grid below.

No partial credit for 1–12.

Questions 13, 14 and 15 are not multiple choice.

	a	b	c	d	e
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					

TA's only

MC_____

13_____

14_____

15_____

TOTAL_____

1. Which of the following is **most correct** concerning the function $y = f(x)$, which is differentiable at a.

   a) $f'(a) = \lim_{\delta \to 1} \dfrac{f(a+\delta) - f(a)}{\delta}$

   b) $f'(a) = \lim_{\delta \to 0} \dfrac{f(a+\frac{1}{\delta}) - f(a)}{\delta}$

   c) $f'(a) = \lim_{\delta \to 0} \dfrac{f(a+\delta) - f(a)}{(\frac{1}{\delta})}$

   d) $f'(a) = \lim_{\delta \to a} \dfrac{f(a+\delta) - f(a)}{\delta}$

   e) $f'(a)$ is the limiting slope of the straight line obtained by zooming in more and more on the graph of $y=f(x)$ at $x=a$.

2. If $f(x) = \sin x \cos x$, $D^2 f(x)$ is

   a) $-\sin x \cos x$

   b) $\sin 2x$

   c) 1

   d) $-4 \sin x \cos x$

   e) $\sin^2 x \cos^2 x$.

3. The position (after t seconds) in meters of a particle travelling on the x axis is $x(t) = 4t^3 - 3t^2 + 2t + 1$. At what time is the acceleration equal to $4\,\mathrm{m/sec^2}$?

   a) 5/12 sec

   b) 7/12 sec

   c) 2.4 sec

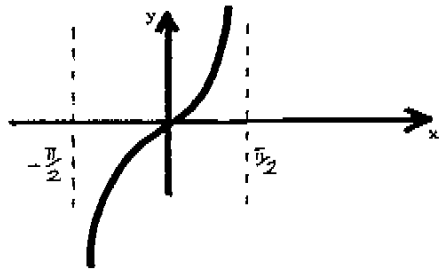
   d) 7/24 sec

   e) 24/7 sec

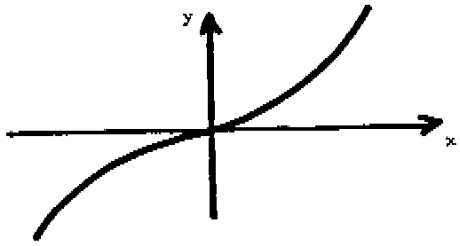
2

4. Which of the following best represents the graph of $f(x) = \sinh x + \tanh x$?

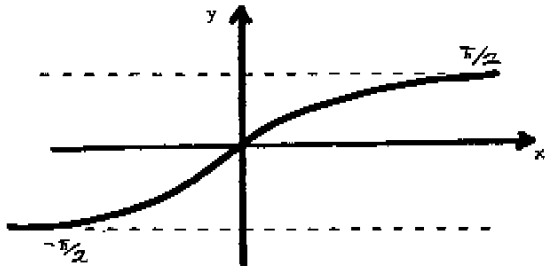
a)



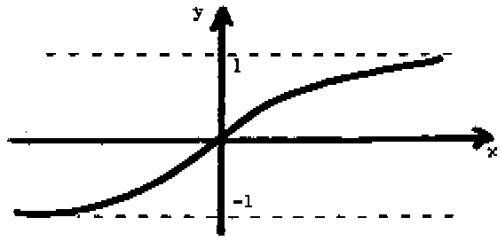
b)



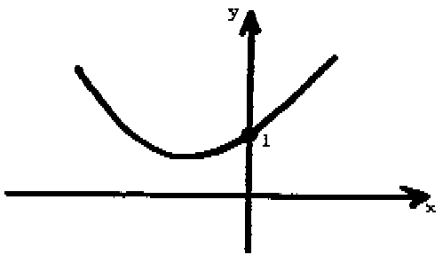
c)



d)



e)



3

5. $D^{51}\cosh x$ is

a) $-\sinh x$

b) $-\cosh x$

c) $\sinh x$

d) $51\sinh^{50} x\cosh x$

e) $51\cosh^{50} x\sinh x$

6. $\lim_{x\to\infty}\frac{\sin x}{x}$ is

a) 1

b) ∞

c) -1

d) 0

e) doesn't exist

7. $\lim_{x\to 1}\frac{\tan^2(x-1)}{(x-1)^2}$ is

a) 1

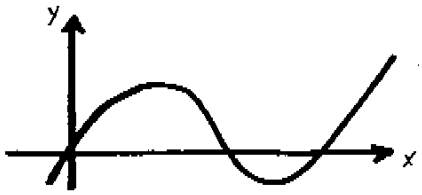
b) ∞

c) -1

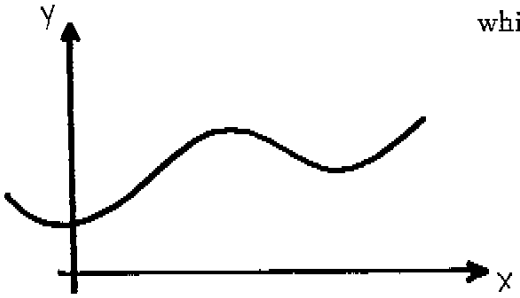
d) 0

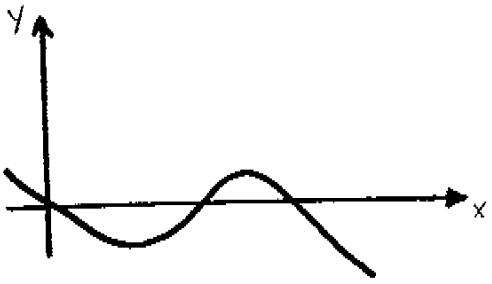
e) doesn't exist

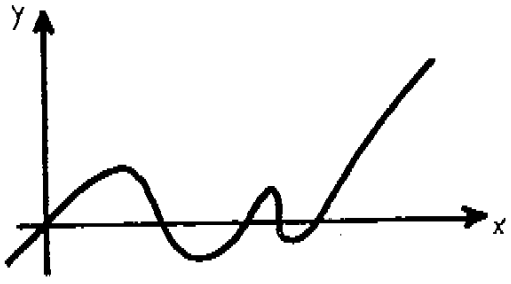
4

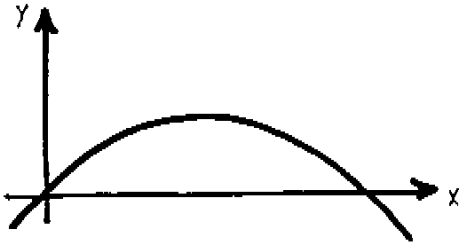
8. If the graph of f' is 

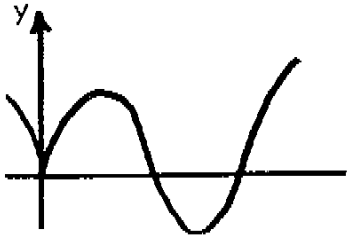
which of the following could be the graph of f?

a) 

b) 

c) 

d) 

e) 

5

9. Which of the following identities is **incorrect?** (For any $a > 0$.)

a) $e^{\ln(a^2)} = a^2$

b) $\ln(e^{a^2}) = a^2$

c) $\ln(\ln(e^{a^2})) = 2\ln(a)$

d) $\ln(\ln(e^{a^2})) = \ln(a^2)$

e) $e^{\ln(2a)} = a^2$

10. Which of the following functions is 1–1?

a) $f(x) = \begin{cases} x^2 & x > 0 \\ -x^2 & x \leq 0 \end{cases}$

b) $f(x) = \sin x \quad \text{for } -\pi < x < \pi$

c) $f(x) = \cosh x \quad \text{for } -\pi/2 \leq x \leq \pi/2$

d) $f(x) = \sinh(x^2) \quad \text{for all } x$

e) $f(x) = \ln(x^4) \quad \text{for } x \neq 0$

11. $\frac{d}{dx}(sec^2 x)$ is

a) $2\tan x \; sec^2 x$

b) $\tan x$

c) $\tan x + C$

d) $\tan^2 x$

e) $-2\tan^3 x$

12. $\frac{d}{dx}(\ln(\ln(\ln(x))))$ is

a) $\ln(\ln(\frac{1}{x}))$

b) $-\frac{2}{x^3}$

c) $[\ln(\ln(x))]^{-1}[\ln(x)]^{-1}x^{-1}$

d) $-\ln(\ln(x))^{-2}x^{-1}$

e) $-\ln(\ln(x))^{-1}x^{-2}$

6

Longer Questions

13. *(30 pts)* Find the slopes of the straight line, or lines, tangent to the curve $y^2 = x - 5$ and passing through the point (2,1).

Write answer here: _____

7

14. *(30 pts)* A man walks along a straight path at a speed of 5 ft/s. A searchlight is located on the ground 12 ft from the path and is kept focused on the man. At what speed is the searchlight moving (in radians/sec) when the man is 16 ft from the point on the path nearest the searchlight?

Write answer here: _____

8

15. *(20 pts)* Use logarithmic differentiation to find $\frac{d}{dx}\left(\sqrt[4]{\frac{x^3-1}{x^2+1}}\right)$

Write answer here: _____

9