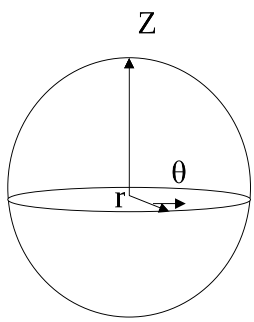
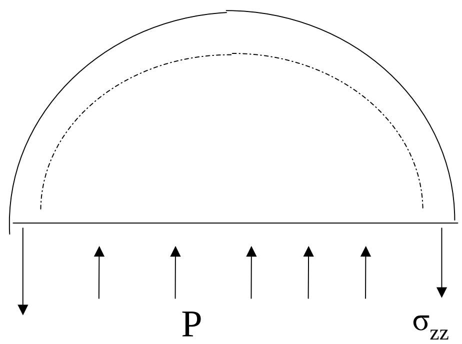
UNIVERSITY OF CALIFORNIA
College of Engineering
Departments of Mechanical Engineering and Materials Science & Engineering

Prof. Ritchie

C-ME 124/C-MSE 113
Mechanical Behavior of Materials
Midterm Exam #1 Solutions





The stresses in the spherical vessel can be found by establishing equilibrium for a section of the sphere as shown above:
$P\pi r^2 = \sigma_{zz} 2\pi r t$, $\sigma_{zz} = Pr/2t$
by symmetry $\sigma_{\theta\theta} = \sigma_{zz} = Pr/2t$
For thin walled pressure vessels $\sigma_{rr} = 0$

1. What internal pressure will cause yielding in the 5mm thick walls if the vessel is made from a carbon steel(uniaxial tensile properties: E=210 GPa, σ_y=450 MPa,σ_u=560 MPa)?

Using the Tresca Criterion:
$\tau_{max} = \tau_y = 450/2 = 225MPa$
i.e.yielding condition
Also $\tau_{max} = \sigma_{\theta\theta}/2 = Pr/4t$
$P_{yield} = 4t\tau_{max}/r = 4.5MPa$

Using the Von Mises Criterion:
$\sigma_y = [0.5[(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2] + 3(\sigma_{12} + \sigma_{23} + \sigma_{31})]^{1/2}$
$\sigma_y = Pr/2t$
$P_{yield} = 2t\sigma_y/r = 4.5MPa$

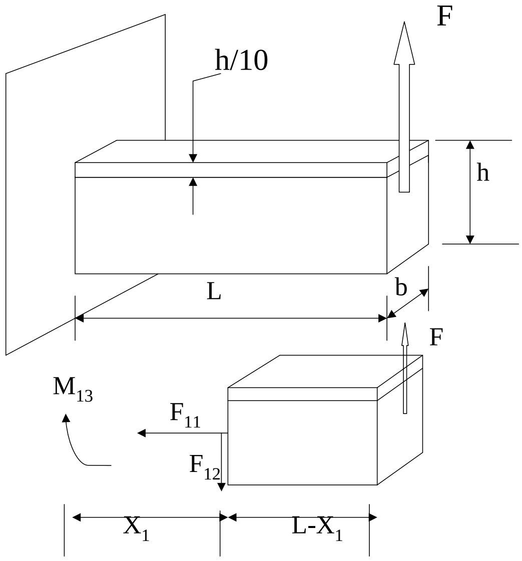
2.What are the principal stresses and the maximum shear stress at the maximum operating pressure of 1800 kPa?
$\sigma_{\theta\theta} = \sigma_{zz} = Pr/2t = 180MPa$
$\sigma_{rr} = 0$
$\tau_{max} = \sigma_{\theta\theta}/2 = 90MPa$

3.What is the factor of safety for the vessel(compare the maximum operating and yielding pressures)?
Factor of Safety = $P_{yield}/P_{op} = 4.5MPa/1800kPa = 2.5$

Problem 2 $A_s = A_h$ gives $r_s = 1.5$"
Torsional Stiffness is given by:
$T/\phi = GJ/L$
$(T_s/\phi_s)/(T_h/\phi_h) = J_s/J_h$ as $G_s = G_h$ and $L_s = L_h$
$(T/\phi)_s/(T/\phi)_h = (\pi(1.5)^4/2)/[\pi(2.5^4 - 2^4)/2] = 0.219$

It is therefore quite clear why automobile manufacturers have employed hollow driveshafts over the years!

Problem 3



Force Equilibrium gives:
$F_{11} = \int \sigma_{11}\, dA. = 0$

Moment Equilibrium gives:
$M_{13} = F(L - x_1) = -\int \sigma_{11} x_2\, dA. = 0$

Let the neutral axis be located at $\bar{x}_2$ from the geometric axis($x_2 = 0$)
$\epsilon_{11} = -(x_2 - \bar{x}_2)/R$, where R is the radius of curvature.

$F_{11} = \int \sigma_{11}\, dA = 0 = \int [-E(x_2 - \bar{x}_2)b/R]\, dx_2 = \int_{-h/2}^{2h/5} [-E_1(x_2 - \bar{x}_2)b/R]\, dx_2 + \int_{2h/5}^{h/2} [-E_2(x_2 - \bar{x}_2)b/R]\, dx_2$

$0 = E_1[4/50h^2 - 2/5h\bar{x}_2 - 1/8h^2 - 1/2h\bar{x}_2] + E_2[1/8h^2 - 1/2h\bar{x}_2 - 4/50h^2 + 2/5h\bar{x}_2]$

$0 = E_1(4/50h^2 - 1/8h^2) + E_2(1/8h^2 - 4/50h^2) - \bar{x}_2[E_1(2/5h + 1/2h) + E_2(1/2h - 2/5h)]$

$\bar{x}_2 = [0.045h(E_2 - E_1)]/(0.9E_1 + 0.1E_2)$
$\bar{x}_2 = -1.918mm$
i.e.the neutral axis is located 1.918mm *below* the geometric axis.

Using the Moment Equilibrium equation:
$M_{13} = -\int \sigma_{11} x_2\, dA = -\int [-E(x_2 - \bar{x}_2)x_2 b/R]\, dx_2 = F(L - x_1)$

$E_1 b/R \int_{-h/2}^{2h/5} (x_2^2 - x_2\bar{x}_2)\, dx_2 + E_2 b/R \int_{2h/5}^{h/2} (x_2^2 - x_2\bar{x}_2)\, dx_2 = F(L - x_1)$

$b/R[E_1(8/375h^3 - 4/50h^2\bar{x}_2 + 1/24h^3 + 1/8h^2\bar{x}_2) + E_2(1/24h^3 - 1/8h^2\bar{x}_2 - 8/375h^3 + 4/50h^2\bar{x}_2)] = F(L - x_1)$

Plugging in $E_1 = 210GPa$,$E_2 = 45GPa$,$h = 0.05m$,$\bar{x}_2 = -1.918mm$,$b = 0.025m$ we get:

$\frac{43230}{R} = F(L - x_1)$

Recall that $\frac{1}{R} = \frac{\partial^2 u}{\partial x_1^2}$, where u is the deflection of the beam.

Integrating once we get:
$\frac{\partial u}{\partial x_1} = \frac{F}{43230}(Lx_1 - \frac{x_1^2}{2} + C_1)$

Boundary Condition:
$\frac{\partial u}{\partial x_1} = 0$ @ $x_1 = 0$ gives $C_1 = 0$

Integrating once again:
$u = \frac{F}{43230}(\frac{Lx_1^2}{2} - \frac{x_1^3}{6} + C_2)$
Boundary Condition:
$u = 0$ @ $x_1 = 0$ gives $C_2 = 0$

Maximum Deflection u_{max} occurs at $x_1 = L$

$u_{max} = \frac{FL^3}{129690}$

If $u_{max} = 3mm$ at $L = 55cm$ then:

$F = \frac{129690(0.003)}{0.55^3}$=2338.5 N