P. Vojta
Fall 2001

Math 1A Second Midterm

50 minutes

1. (14 points)

   (a). Find $\lim\limits_{x \to 0+} (\sin x)^x$.

   (b). Find $\lim\limits_{x \to \infty} \left(\dfrac{\pi}{2} - \arctan x \right)^x$.

   (c). Find $\lim\limits_{x \to \infty} \dfrac{\cosh^{-1} x}{\ln x}$.

2. (20 points)

   (a). *Without using l'Hospital's rule,* find $\lim\limits_{x \to 0} \dfrac{\sin(\sin x)}{\sin \pi x}$.

   (b). Find $\dfrac{d}{dx} \arcsin x$.

   (c). Find $\dfrac{d}{dx}\left(\cos\left(e^{x^2}\right)\right)$.

3. (9 points) Find $\dfrac{d}{dx}\left(x^{(e^x)}\right)$.

4. (12 points) Let $f(x)$ be a function such that:

   (1) its domain is $(-\infty, 0) \cup (0, 1) \cup (2, \infty)$;

   (2) $f'(x) = 0$ everywhere on the domain of f; and

   (3) $f(.5) = 3$ and $f(3) = 7$.

   Say as much as you can about $\lim\limits_{x \to -\infty} f(x)$ and $\lim\limits_{x \to \infty} f(x)$.

5. (20 points) A rectangular playground is to be fenced off and divided in half by another fence parallel to one side of the playground. The total area of both halves is to be 600 square feet. Find the dimensions of the playground that will use the minimal amount of fencing.

6. (25 points) Graph the function
$$f(x) = e^{1/x} \ .$$

List all features of the graph.
You may use the next page for additional space.
[Axes for graphing were provided on the next page.]