Math 1A Midterm 1 2005-9-29 11:00-12:30pm.   R. Borcherds

You are allowed 1 sheet of notes. Calculators are not allowed. Each question is worth 3 marks, which will only be given for a clear and correct answer.

1. Find the domain of the function $g(u) = \sqrt{u} + \sqrt{2 - u}$.

2. Sketch the graph of $y = |x^2 - 2x|$.

3. Find a formula for the inverse of the function $f(x) = 1 + e^{x^3}$.

4. Sketch the graph of a function f that satisfies the conditions
$$\lim_{x \to 0^-} f(x) = 1, \ \lim_{x \to 0^+} f(x) = -1, \ \lim_{x \to 1^-} f(x) = 1, \ \lim_{x \to 1^+} f(x) = -1, f(2) = 1.$$

5. Evaluate the limit
$$\lim_{x \to 1} \frac{x^3 - 1}{x^2 - 1}$$

6. How close to 2 do we have to take x so that $5x + 3$ is within a distance of 0.01 from 13?

7. Find the numbers at which f is discontinuous, where f is defined by $f(x) = x^2$ if $x \leq 1$, $f(x) = 1/x$ if $1 < x < 3$, $f(x) = 1/2 + \sqrt{x - 3}$ if $x \geq 3$.

8. What is
$$\lim_{x \to -\infty} \frac{(3x + 1)(4x + 1)}{(x + 1)(2x + 1)}$$

9. A curve has equation $y = f(x)$. Write an expression for the slope of the secant line through the points $(3, f(3))$ and $(x, f(x))$, and write an expression for the slope of the tangent line at $(3, f(3))$.

10. If $g(x) = x^3 + x^2 + x + 1$, find $g'(0)$ and use it to find an equation of the tangent line to the curve $y = x^3 + x^2 + x + 1$ at the point $(0, 1)$.

11. Sketch the graph of the function $y = x^2|x|$, say where it is differentiable, and find a formula for its derivative.

12. Differentiate the function $y = e^{x+1} + 4\pi^2 + (x + 1)/\sqrt{x}$

13. At what point on the curve $y = 1 + 2e^x - 3x$ is the tangent line parallel to the line $3x - y = 5$?

14. Differentiate $xe^x(\sqrt{x} + 1)$

15. Differentiate
$$\frac{x^2}{1 + 1/x}$$