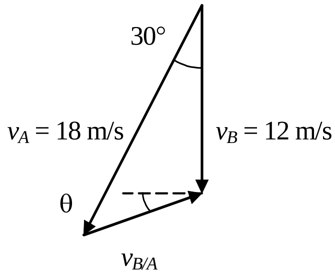
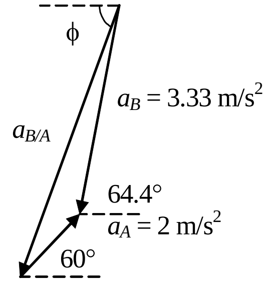
Problem 1.





(a) Since the direction of car A is constant, a coordinate system attached to A will only be translating. In the vector equation

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

only the vector $\mathbf{v}_{A/B}$ is not known. Either from a graphical solution or trigonometry,

$$v_{B/A}^2 = v_B^2 + v_A^2 - 2v_B v_A \cos 30° = 324 + 144 - 2(18)(12)\cos 30°$$

$$\Rightarrow \qquad v_{B/A} = 9.69 \text{ m/s}$$

With an additional application of the law of cosines,

$$\theta = 21.74°$$

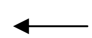
In a similar fashion,

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

Observe that

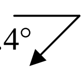
$$(a_B)_t = 3 \text{ m/s}^2$$

$$(a_B)_n = \frac{v_B^2}{\rho} = \frac{144}{100} = 1.44 \text{ m/s}^2$$



Thus

$$a_B = 3.33 \text{ m/s}^2$$

64.4° 

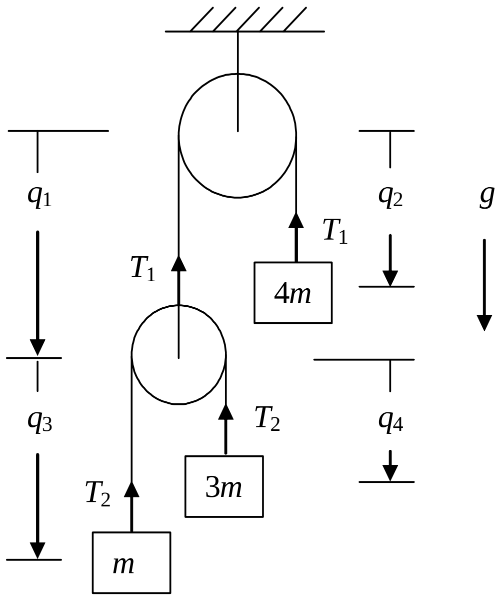
Either from a graphical solution or trigonometry,

$$a_{B/A} = 5.32 \text{ m/s}^2$$

$$\phi = 62.72°$$

(b) A coordinate system attached to B is a rotating system. For example, a set of rectangular coordinates attached to B with the y-axis aligned with the velocity of B is continuously changing its direction. Thus the acceleration of car A as observed from car B is not equal to $-\mathbf{a}_{B/A}$.

Problem 2.



(a) Let T_1 be the tension in the upper string and T_2 tension in the lower string. It is obvious that $T_1 = 2T_2$. A force balance on each mass gives

$$4mg - T_1 = 4m\ddot{q}_2 \tag{1}$$

$$mg - T_2 = m(\ddot{q}_1 + \ddot{q}_3) \tag{2}$$

$$3mg - T_2 = 3m(\ddot{q}_1 + \ddot{q}_4) \tag{3}$$

Observe that $\ddot{q}_1 = -\ddot{q}_2$ and $\ddot{q}_3 = -\ddot{q}_4$. It follows that the above three equations involve only three unknowns $\ddot{q}_2$, $\ddot{q}_3$ and T_2. Solution yields

$$\ddot{q}_2 = \frac{1}{7}g = 1.40\,\mathrm{m/s^2} \quad \downarrow$$

(b) Since the masses *m* and 3*m* connected to the lower pulley are in motion, forces in the system are not in equilibrium. The tension in the upper string is

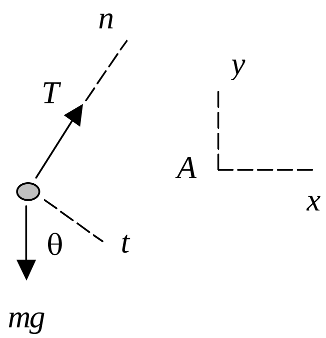
$$T_1 = 4mg - 4m\ddot{q}_2 = \frac{24}{7}mg < 4mg$$

Thus the mass 4*m* has a downward acceleration even when the total mass on each side of the upper pulley is the same.

(c) Velocity of mass 4*m* after 2 s is directed downward and is equal to

$$v = v_0 + at = \frac{2}{7}g = 2.80\,\mathrm{m/s} \quad \downarrow$$

Problem 3.



(a) Let the rope break in position $\theta = \alpha$. Before the rope breaks, the bag travels in a circle of radius *l*. In any position $\theta \le \alpha$,

$$\sum F_t = ma_t \quad \Rightarrow \quad mg\cos\theta = ma_t \quad \Rightarrow \quad \dot{v} = g\cos\theta \qquad (1)$$

$$\sum F_n = ma_n \quad \Rightarrow \quad T - mg\sin\theta = m\frac{v^2}{l} \qquad (2)$$

From kinematics and equation (1),

$$\dot{v} = \frac{dv}{d\theta}\dot{\theta} = \frac{dv}{d\theta}\frac{l\dot{\theta}}{l} = \frac{vdv}{ld\theta} = g\cos\theta \quad \Rightarrow \quad \int_0^v vdv = \int_0^\theta g\cos\theta\, ld\theta$$

$$\Rightarrow \quad v^2 = 2gl\sin\theta \qquad (3)$$

When $\theta = \alpha$, $T = 2mg$. It follows from equation (2) that

$$2mg - mg\sin\alpha = m\frac{2gl\sin\alpha}{l} \quad \Rightarrow \quad \alpha = \sin^{-1}\frac{2}{3} = 41.81^\circ$$

(b) Set up a rectangular system with origin at *A*. When the rope breaks, the position of the bag is

$$(x_0, y_0) = (l - l\cos\alpha, -l\sin\alpha) = (2.546, -6.667)$$

In that position, $v = \sqrt{2gl\sin\alpha} = 11.431$ and the distance to fall before reaching the level *C* is $h - |y_0| = h - 6.667 = 23.333$. Along the *y*-direction,

$$y = v_y t - \frac{1}{2}gt^2 \quad \Rightarrow \quad -23.333 = (-11.431\cos\alpha)t - \frac{1}{2}gt^2$$

$$\Rightarrow \quad t = 1.479$$

Along the *x*-direction,

$$x = v_x t = 11.431\sin\alpha(1.479) = 11.271$$

Thus the horizontal distance of *C* from *A* is $11.27 + x_0 = 11.27 + 2.55 = 13.82\,\text{m}$