

Stat 21 Fall 2003
Midterm 1
 Open book, 50 minutes

Show all work for full credit. Keep 3 decimal places in calculations and answers.
 Frame your answers

Problem 1 (35 minutes, 35 points)

The following data are obtained for about 1000 men:

average height: 69 inches, SD: 2.5 inches
 average forearm length: 18 inches, SD: 1 inch, $r = 0.8$

(assume football shape)

- Of the men who were 6 feet tall, about what percentage had forearms shorter than 18 inches?
- Of men who had 19 inches forearms, about what percentage had height taller than 69.5 inches?
- A man's height is the 96th percentile of height distribution. Predict the percentile rank of his forearm.

a) 6ft. = 72 in. $\frac{72-69}{2.5} = 1.2$ SD away new SD = $\sqrt{1-.8^2} \cdot 1 = .6$

~~new~~ SD for forearm = $1.2 \cdot .8 = .96 \approx .95$ SD

$.96 \cdot 1 + 18 \text{ in} = 18.96 \text{ in}$ (new mean)

$\frac{18-18.96}{.6} = -1.6 \approx -1.60$ for SD

$z = \frac{x-\bar{x}}{SD}$

Answer

$\leftarrow 50 - \frac{89.04}{2} = 5.480\%$ have forearms shorter than 18 in. at 6ft tall

b) $\frac{19-18}{1} = 1$ SD away

new mean = $.8 \cdot 1 \cdot 2.5 + 69 = 71 \text{ in.}$

new SD = $\sqrt{1-.8^2} \cdot 2.5 = 1.5 \text{ in}$

SD = $\frac{69.5-71}{1.5} = -1$

SD = $\frac{68.27}{2} = 15.865$

$100 - 15.865 = 84.135\%$ are taller than 69.5 in. with forearms of 19 in.

c) $(100-96) \cdot 2 = 8$ $100-8 = 92 \approx 1.75$ SD away

new SD for x is $1.75 \cdot .8 = 1.4$ SD

$50 + \frac{83.85}{2} = 91.925$ percentile

$$r = \frac{17.3 - 1.8}{1.5} = .86667$$

$$r = \frac{\text{COV}(x,y)}{SD_x \cdot SD_y} = \frac{74}{6} = 12.3$$

Problem 2 (15 minutes, 15 points)

A. What is the correlation coefficient for the data below (show all work)

x	y	xy	\bar{x}	\bar{y}
0	1	0	1	8
0	3	0		
1	7	7		
1	9	9		
1	13	13		
3	15	45		

$$\frac{0 \cdot 2 + 1 \cdot 3 + 3}{6}$$

$$\frac{1 + 3 + 7 + 9 + 13 + 15}{6}$$

$$SD_x = \sqrt{\frac{(0-1)^2 + (0-1)^2 + 0 + (2-1)^2}{6}}$$

$$= 1$$

$$SD_y = \sqrt{\frac{(8-1)^2 + (3-8)^2 + (7-8)^2 + (9-8)^2 + (13-8)^2 + (15-8)^2}{6}} = 5$$

B. If possible fill in the two blanks so that the correlation for the data below equals the correlation in part A. If this is not possible, explain why.

x	y
-0	-3
-0	-7
-2	-15
-2	-19
-2	-27
-6	-31

(assume -0 is 0)

$$r = \frac{\text{COV}(x,y)}{SD_x \cdot SD_y}$$

The values are $([-0, -7], [-2, -19])$

it is a transformation of $x \cdot (-2) - 1$

$$\bar{x} = \frac{-0 + 0 - 2 - 2 - 2 - 6}{6} = -2$$

$$\bar{y} = \frac{-3 - 7 - 15 - 19 - 27 - 31}{6} = -17$$

$$SD_x = \sqrt{\frac{2^2 + 2^2 + 0^2 + 0^2 + 0^2 + 4^2}{6}} = 2$$

$$SD_y = \sqrt{\frac{14^2 + 10^2 + 2^2 + 2^2 + 10^2 + 14^2}{6}} = 10$$

$$\sum xy = 0 + 0 + 30 + 38 + 54 + 186 = 308$$

$$\frac{\sum xy}{6} = 51.3$$

$$r = \frac{51.3 - (-2) \cdot (-17)}{2 \cdot 10} = .86$$

same correlation