SOLUTION

Stat 21 --- Fall 2003
Midterm 2
Open book --- 50 minutes

Show all work to salvage points. Frame your answers. Show 3 decimal places in your answers. It is not necessary to use the continuity correction, or to do interpolations in the normal table. "Non-negative" means larger than or equal to zero, "positive" means strictly larger than zero.

Problem 1. *(25 minutes, 25 points)*
A box has 2 tickets labeled "-1", 2 ticket labeled "0", and 6 tickets labeled "2":

-1 -1 0 0 2 2 2 2 2 2

A. I draw 5 tickets with replacement. What is the chance of obtaining 3 non-negative tickets?
B. I draw 5 tickets without replacement. What is the chance of obtaining 3 non-negative tickets?
C. I draw 100 tickets with replacement. What is the chance that the number of non-negative tickets will be larger than 74?
D. I draw 100 tickets with replacement. What is the chance that the sum of draws of the positive tickets will be larger than 105?
E. Now suppose the box has 200 tickets labeled "-1", 200 tickets labeled "0", and 600 tickets labeled "2". I draw 100 tickets without replacement. What is the chance that the sum of draws of the positive tickets will be larger than 105?

A. Box is [0 0 1 1 1 1 1 1 1 1] $\binom{5}{3}(0.8)^3(0.2)^2 = 0.205$ or **20.5%**

B. Same box as in A $\binom{5}{3}\frac{8}{10}\times\frac{7}{9}\times\frac{6}{8}\times\frac{2}{7}\times\frac{1}{6} = 0.222$ or **22.2%**

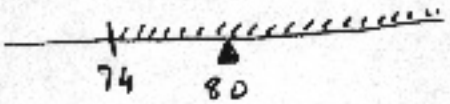
C. Same box as in A. Box average = 0.8

Box SD $= (1-0)\sqrt{\frac{8}{10}\times\frac{2}{10}} = 0.4$

Number of non-negative tickets equal sum of draws

EV SOD $= 100\times 0.8 = 80$ SE SOD $= \sqrt{100}\times 0.4 = 4$

z of 74 $= \frac{74-80}{4} = -1.5$ $50 + \frac{86.64}{2} =$ **93.32%**



D. Box is [0 0 0 0 2 2 2 2 2 2] Box ave = 1.2

Box SD $= (2-0)\sqrt{\frac{4}{10}\times\frac{6}{10}} = 0.9798$ EV SOD $= 100\times 1.2 = 120$

SE SOD $= \sqrt{100}(0.9798) = 9.798$ z of 105 $= \frac{105-120}{9.798} = -1.53$

$50 + \frac{87.99}{2} =$ **93.945%**

E. cf $= \sqrt{\frac{1000-100}{1000-1}} = 0.9492$ SE SOD $= 0.9492\sqrt{100}(0.9798) = 9.30$ $z = \frac{105-120}{9.30} = -1.61$

$50 + \frac{89.04}{2} =$ **94.52%**

(question E continued)

Problem 2. *(25 minutes, 25 points)*
A box has 1000 tickets. Values of these tickets are unknown. I draw without replacement 100 tickets and find 20 "-2", 40 "0" and 40 "3":

-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3

A. Obtain a 95% confidence interval for p, the proportion of non-negative tickets in the box.
(Note: You are required to show the AOD, the SE, and the error margin. Give the answer in the form: p equals ... give or take ... at 95% confidence)
B. Obtain a 99% confidence interval for a, the average value of all positive tickets in the box.
(Note: You are required to show the AOD, the SE, and the error margin. Give the answer in the form: a equals ... give or take ... at 99% confidence)

A. All negative tickets are replaced by "0", on non negative tickets by "1". Sample has 20 "0" and 80 "1".

AOD = 0.8 Data SD = $(1-0)\sqrt{0.8 \times 0.2} = 0.4$

SE = $\frac{0.4}{\sqrt{100}} = 0.04$ Error = $1.96 \times 0.04 = 0.0784$

Answer: $p = 80\% \pm 7.84\%$ at 95% confidence

B. All negative tickets are replaced by "0", all positive tickets are unchanged. Sample has 60 "0" and 40 "3".

AOD = $\frac{40 \times 3}{100} = 1.2$ Data SD = $(3-1)\sqrt{0.6 \times 0.4} = 1.470$

SE = $\frac{1.470}{\sqrt{100}} = 0.147$ Error = $2.575 \times 0.147 = 0.379$

Answer: $a = 1.2 \pm 0.379$ at 99% confidence