LAST Name ——————————————— FIRST Name ———————————

Discussion Time ———————————

- **(10 Points)** Print your name and discussion time in legible, block lettering above AND on the last page where the grading table appears.

- This exam should take up to 90 minutes to complete. You will be given at least 90 minutes, up to a maximum of 110 minutes, to work on the exam.

- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except one double-sided $8.5" \times 11"$ sheets of handwritten notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff— *including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.

- **The exam printout consists of pages numbered 1 through 6.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the six numbered pages. If you find a defect in your copy, notify the staff immediately.

- Please write neatly and legibly, because *if we can't read it, we can't grade it.*

- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*

- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.

- We hope you do a *fantastic* job on this exam.

MT2.1 (35 Points) We form a system H by placing a pair of discrete-time LTI systems F and G in cascade, as shown below.
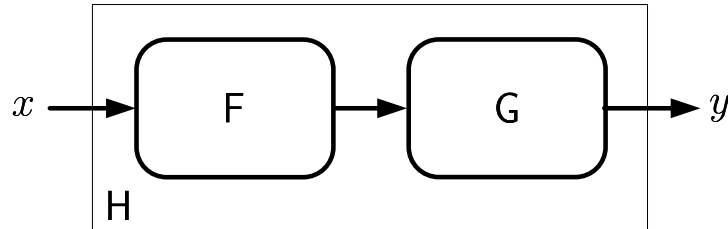
Let f, g, and h denote the impulse responses of the systems F, G, and H, respectively. Moreover, $\widehat{F}$, $\widehat{G}$, and $\widehat{H}$ are the transfer functions of the systems F, G, and H, respectively.

The impulse responses and transfer functions of the three systems are described below:

$$f(n) = \begin{cases} 1 & n = 0, 1, 2 \\ 0 & \text{elsewhere.} \end{cases} \qquad \longleftrightarrow \qquad \widehat{F}(z) = 1 + z^{-1} + z^{-2}$$

$$g(n) = \begin{cases} +1 & n = 0, 3, 5, 8 \\ -1 & n = 1, 4, 7 \\ 0 & \text{elsewhere.} \end{cases} \qquad \longleftrightarrow \qquad \widehat{G}(z) = 1 - z^{-1} + z^{-3} - z^{-4} + z^{-5} - z^{-7} + z^{-8}$$

$$h(n) = \begin{cases} 1 & n = 0, 5, 10 \\ 0 & \text{elsewhere.} \end{cases} \qquad \longleftrightarrow \qquad \widehat{H}(z) = 1 + z^{-5} + z^{-10}.$$

(a) (10 Points) Determine the region of convergence of $\widehat{G}(z)$, the transfer function of the subsystem G.

(b) (20 Points) Show that the frequency response of the subsystem G can be expressed as
$$G(\omega) = A(\omega)\, e^{i\alpha\omega},$$
where $A(\omega)$ is a real-valued *amplitude* (not necessarily magnitude) function, and α is a constant. Determine a reasonably simple expression for $A(\omega)$ and the numerical value of α.

(c) (25 Points) Provide a well-labeled pole-zero diagram for $\widehat{G}(z)$. To help you plot, you might want to know that $2\pi/15$ radians is approximately 24 degrees.

(d) (15 Points) If the input to the cascade interconnection H is the signal x described by

$$\forall n \in \mathbb{Z}, \quad x(n) = \sum_{\ell=-\infty}^{+\infty} \delta(n - 15\ell),$$

determine and provide a well-labeled plot of the corresponding output signal y.

(e) (15 Points) Express the frequency response $H(\omega)$ of the cascade interconnection H in terms of the frequency response $F(\omega)$ of the subsystem F.

(f) (20 Points) Provide well-labeled plots of $|F(\omega)|$ and $\angle F(\omega)$, the magnitude and phase responses, respectively, of the subsystem F. Determine whether F is a low-pass filter, high-pass filter, band-pass filter, comb filter, notch filter, anti-notch filter, or some other type of filter.

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Problem	Points	Your Score
Name	10	
1(a)	10	
1(b)	20	
1(c)	25	
1(d)	15	
1(e)	15	
1(f)	20	
Total	**115**	