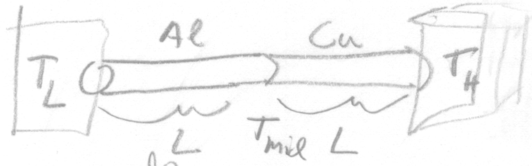
1a)



$\frac{dQ_1}{dt} = \frac{dQ_2}{dt}$ → +3

$$\frac{dQ_1}{dt} = \frac{dQ_2}{dt}$$

$$k_{Al}\frac{A}{L}(T_{mid} - T_L) = k_{cu}\frac{A}{L}(T_H - T_{mid})$$

$$T_{mid}(k_{Al} + k_{cu}) = k_{cu}T_H + T_L k_{Al} \Rightarrow +2$$

$$T_{mid} = \frac{k_{cu}T_H + k_{AL}T_L}{k_{Al} + k_{cu}}$$

$$k_{Al} = k_{cu} = k$$

$$T_{mid} = \frac{k(T_H + T_L)}{2k} = \frac{T_H + T_L}{2}$$ ⇒ this makes sense, average of the two T's

→ +2.5

$$k_{Al} = \frac{1}{2}k_{cu}$$

$$T_{mid} = \frac{k_{cu}T_H + \frac{1}{2}k_{cu}T_L}{\frac{3}{2}k_{cu}} = \frac{2}{3}\left(T_H + \frac{1}{2}T_L\right)$$

$$= \frac{2}{3}T_H + \frac{1}{3}T_L \Rightarrow +2.5$$

⇒ k_{cu} more conductive than k_{Al} so T_{mid} should be higher than before.

Problem 1b)

The spherical solid has mass $m = \rho\frac{4\pi R^3}{3}$ and surface area $A = 4\pi R^2$. Assume the temperature of the space is effectively 0 K. According to the Stefan-Boltzmann equation,

$$\frac{dQ}{dt} = -e\sigma AT^4 \tag{1}$$

where σ is the Stefan-Boltzmann constant. On the other hand,

$$Q = mC\Delta T$$

$$\frac{dQ}{dt} = mC\frac{dT}{dt} \tag{2}$$

Combine (1) and (2),

$$-e\sigma AT^4 = mC\frac{dT}{dt}$$

$$\int_{ti}^{t_f} dt = \frac{-mC}{e\sigma A}\int_{T_0}^{T_1} \frac{dT}{T^4}$$

Therefore

$$t_f - t_i = \frac{-mC}{e\sigma A}(\frac{-1}{3T_1^3} + \frac{1}{3T_0^3})$$

Since $\frac{m}{A} = \frac{1}{3}R\rho$,

$$t_f - t_i = \frac{\rho RC}{9e\sigma}(\frac{1}{T_1^3} - \frac{1}{T_0^3}) \tag{3}$$

which is the time it takes to cool down to temperature T_1.

Midterm 1 Problem 1c Solution

Assuming the gas is a monatomic ideal gas and using the equipartition theorem,

$$\overline{KE} = \frac{1}{2}mv_{rms}^2 = \frac{3}{2}k_bT$$

$$\Rightarrow v_{rms} = \sqrt{\frac{3k_bT}{m}} \tag{1}$$

Then using the ideal gas law to express k_bT in terms of pressure and volume:

$$k_bT = \frac{PV}{N}$$

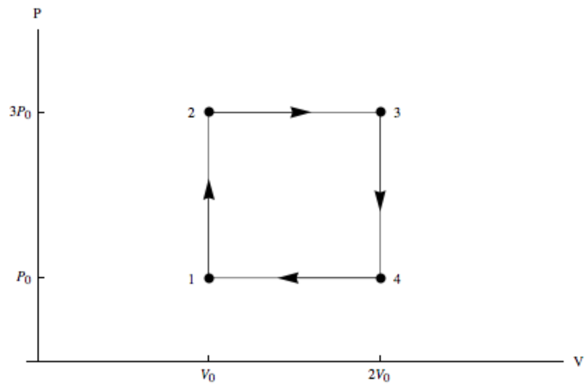
$$\Rightarrow v_{rms} = \sqrt{\frac{3PV}{Nm}} \tag{2}$$

The question states that r is the gas density - the total mass of the gas divided by the volume. Since there are N atoms that each weigh m:

$$r = \frac{Nm}{V} \tag{3}$$

$$\Rightarrow v_{rms} = \sqrt{\frac{3P}{r}} \tag{4}$$

Problem 2: Heat Engine (Giancoli 20-69 4th ed.)



a. We can figure out when heat is flowing in by using the First Law of Thermodynamics in the form $Q_{\text{in}} = \Delta E + W$. If Q_{in} is positive during some part of the cycle, then that part of the cycle contributes to Q_{H}. From this rule we see that heat flows in during the processes $1 \to 2$ and $2 \to 3$.

To calculate those contributions to Q_H, we first calculate the changes in internal energy and the work done by the gas during each of the processes, and then plug those into the First Law:

$$Q_H^{1\to2} = \Delta E^{1\to2} + W^{1\to2} = \frac{3}{2}(3P_0V_0 - P_0V_0) + 0 = \boxed{3P_0V_0}\,, \tag{1}$$

$$Q_H^{2\to3} = \Delta E^{2\to3} + W^{2\to3} = \frac{3}{2}(6P_0V_0 - 3P_0V_0) + 3P_0V_0 = \boxed{\tfrac{15}{2}P_0V_0}\,. \tag{2}$$

The total Q_H is just the sum of these two contributions:

$$Q_H = \boxed{\tfrac{21}{2}P_0V_0}\,. \tag{3}$$

b. The total work done by the engine is equal to the area inside the cycle on the PV diagram:

$$W_T = \boxed{2P_0V_0}\,. \tag{4}$$

The efficiency is now easily calculated from the usual formula:

$$e = \frac{W_T}{Q_H} = \boxed{\tfrac{4}{21}}\,. \tag{5}$$

This is about 19%.

c. To calculate the Carnot efficiency, we first need to figure out the extreme temperatures T_H and T_L. The ideal gas law tells us that $T = PV/Nk$ at any point in the cycle. We see that the lowest temperature is $T_L = P_0V_0/Nk$, which occurs at point 1, and the highest temperature is $T_H = 6P_0V_0/Nk$, which occurs at point 3. Now we just plug this into the Carnot efficiency equation:

$$e_{\text{Carnot}} = 1 - \frac{T_L}{T_H} = 1 - \frac{1}{6} = \frac{5}{6}\,. \tag{6}$$

This is about 83%. The rectangular engine has $\boxed{\text{0.23 times the efficiency}}$ of the Carnot engine.

Grading Rubric for Problem 2: Heat Engine (Giancoli 20-69 4th ed.)

a. [10 points]

- 2 points: Identify the parts of the cycle where heat flows in.
- 6 points: Compute the heat flow into the gas during relevant parts of the cycle (partial credit if the computation is correct but the wrong parts of the cycle were chosen).
- 2 points: Add up the individual heat flows to get the total heat flow into the engine.

b. [10 points]

- 7 points: Compute the total work done by the engine (partial credit if work is computed correctly for some parts of the cycle, but not for the cycle as a whole).
- 3 points: Compute the efficiency of the engine using the answer from part (a) (partial credit for having the correct formula in terms of work and heat).

c. [10 points]

- 8 points: Compute highest and lowest temperatures (partial credit for identifying where the highest and lowest temperatures occur; partial credit for a correct temperature calculation even if it is not the highest/lowest that the engine acheives).
- 2 points: Compute the Carnot efficiency and note that it is bigger than that of the rectangle (partial credit for knowing that the Carnot engine should be more efficient).

3) a) S is a state variable. After each complete cycle, the gas returns to its original state, so $\Delta S_{gas} = 0$

Since the carnot cycle is reversible, we know that

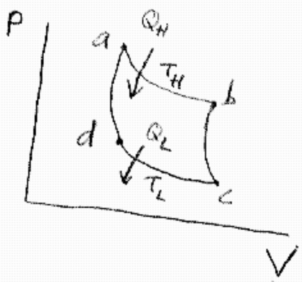
$$\Delta S_{universe} = \Delta S_{gas} + \Delta S_{environment} = 0$$

So, $0 + \Delta S_{environment} = 0$

$$\Rightarrow \Delta S_{envirsoment} = 0$$

b) $e = 1 - \frac{Q_L}{Q_H} \overset{\text{carnot}}{=} 1 - \frac{T_L}{T_H}$

So, $\frac{Q_L}{T_L} = \frac{Q_H}{T_H}$.



$$\Delta S_{a\to b} = + \frac{Q_H}{T_H}$$

$$\Delta S_{b\to c} = 0$$

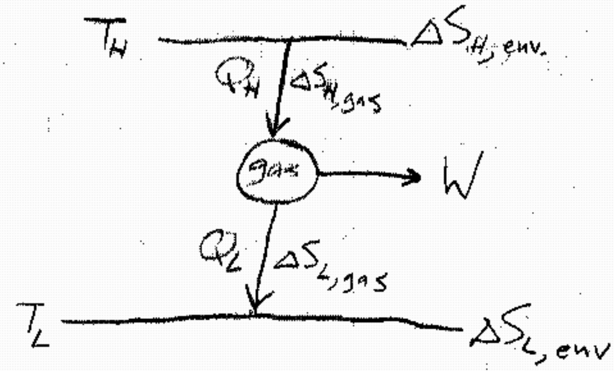
$$\Delta S_{c\to d} = - \frac{Q_L}{T_L}$$

$$\Delta S_{a\to a} = 0$$

So $\Delta S_{gas, total} = \frac{Q_H}{T_H} - \frac{Q_L}{T_L} = 0$.

Reversible, so $\Delta S_{environment} = -\Delta S_{gas} = 0$.
Thus, $\Delta S_{universe} = 0$.

Or



reversible, so $\Delta S_{H,env.} = -\Delta S_{H,gas}$

$$\Delta S_{L,env.} = -\Delta S_{L,gas}$$

Thus, $\Delta S_{universe} = \Delta S_{gas} + \Delta S_{environment}$

$$= (\Delta S_{H,gas} - \Delta S_{L,gas}) - (\Delta S_{H,gas} - \Delta S_{L,gas})$$

$$= 0$$