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Math 54 Second Midterm Fall 2009 Instructor: D.-V. Voiculescu

This is a "closed book" exam, so you may not bring in or use notes or the textbook. Calculators are not allowed.

Please write your name, SID and Discussion Section # on everything you hand in, including this sheet of paper on which you have to provide answers to Problem II (the true or false questions). For Problem I you must show the method and calculations you use to get the answers (write the solutions to the questions in Problem I in your blue book). The Requirement is 20 points.

Problem I (3+4+4 pts) Consider the following two ordered bases in $\mathbb{R}^4$

$B = \{(0,0,1,0),(0,1,1,0),(1,1,1,0),(1,1,1,1)\}$ $\{b_1, b_2, b_3, b_4\}$

$C = \{(0,0,0,1),(1,0,0,1),(0,1,1,0),(1,1,0,1)\}$ $\{c_1, c_2, c_3, c_4\}$

a) Find the transition matrix from C to B.

b) Find the orthonormal basis D = {x,y,z,t} which is obtained from C by the Gram-Schmidt procedure.

c) Find the 4x4 matrix T with eigenvectors x,y,z,t for the respective eigenvalues 1,-1,2,0.

Problem II (9 pts, each question 1 pt). Check True or False.

	True	False
a) All solutions of $y'' + y = 0$ are bounded.	✓	
b) $y'' - 3y' + 5y = 0$ has two linearly independent solutions $y_1(t)$, $y_2(t)$ on $\mathbb{R}$ with Wronskian at zero $W[y_1, y_2](0)=3$.	✓	
c) The function y(t) which is $= e^t$ if $t>0$ and $=e^{2t}$ if $t\leq 0$, is a solution of $y'' - 3y' + 2y = 0$ on $\mathbb{R}$.		✓
d) $e^{(1+i)\pi}$ is a real number.	✓	
e) $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}$ is a diagonalizable matrix	✓	
f) The matrix $\begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ is diagonalizable		✓
g) The transpose of a symmetric matrix is equal to its inverse.		✓
h) $\begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 0 & -1 & 2 \end{pmatrix}$ is an orthogonal matrix. Not normalized!		✓
i) If A is an orthogonal matrix then A^T is always invertible.	✓	

orthogonal matrix true.

$A^T = A^{-1}$ yes.

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