CS 70 Discrete Mathematics and Probability Theory

Fall 2010 Tse/Wagner MT 2 Soln

Problem 1. [Rolling Dice] (25 points)

You roll a fair die three times. Consider the following events:

$$
\begin{aligned}
A &= \text{first roll is a 3} \\
B &= \text{second roll is a 4} \\
C &= \text{second roll is a 3} \\
D &= \text{the first roll is a 3 and the second roll is a 4} \\
E &= \text{the sum of the first and second roll is 5} \\
F &= \text{the sum of the first and second roll is 3} \\
G &= \text{the sum of the second and third roll is 6} \\
H &= \text{the first roll is different from the second roll} \\
I &= \text{the first roll is different from the third roll}
\end{aligned}
$$

(a) TRUE or $\boxed{\text{FALSE}}$: A and B are disjoint.

(b) TRUE or $\boxed{\text{FALSE}}$: A and C are disjoint.

(c) TRUE or $\boxed{\text{FALSE}}$: $\Pr[A] < \Pr[D]$

(d) TRUE or $\boxed{\text{FALSE}}$: A and E are independent.

(e) TRUE or $\boxed{\text{FALSE}}$: A and F are independent.

(f) $\boxed{\text{TRUE}}$ or FALSE: A and H are independent.

(g) TRUE or $\boxed{\text{FALSE}}$: H and I are disjoint.

(h) $\boxed{\text{TRUE}}$ or FALSE: H and I are independent.

Comment: $\Pr[H] = 5/6$, $\Pr[I] = 5/6$, and $\Pr[H \cap I] = 25/36$, so $\Pr[H \cap I] = \Pr[H] \times \Pr[I]$.

Grading: As stated in the exam, parts (a)-(h) are worth 2 points each, with 1 point awarded for each part left blank, 0 for each incorrect answer.

(i) (2 points) Compute $\Pr[E \cap G]$.

The outcomes in $E \cap G$ are $(1,4,2)$, $(2,3,3)$, $(3,2,4)$, $(4,1,5)$, where (x,y,z) means the value of the three rolls were x,y,z, respectively. Each outcome has probability $1/6^3$. Therefore, $\Pr[E \cap G] = 4/6^3$.

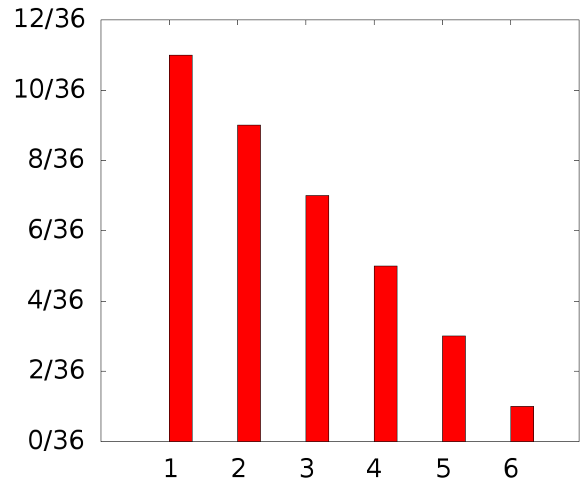
(j) (3 points) Compute $\Pr[B|G]$.

> $\Pr[B|G] = \Pr[B \cap G]/\Pr[G] = (1/36)/(5/36) = 1/5$, since there is 1 outcome in $B \cap G$ (namely, $(4,2)$) and 5 outcomes in G (namely, $(i, 6-i)$ for $i = 1,2,3,4,5$) and all 36 outcomes are equally likely.

Grading: We did not award partial credit for parts (i) or (j).

(k) (4 points) Let X be the value of the first roll and Y be the value of the second roll. Compute and plot the distribution of $Z = \min(X,Y)$, i.e., Z is the minimum of X and Y.

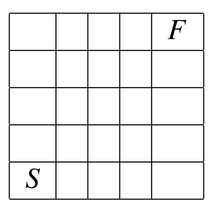
$$\Pr[Z=1] = \frac{11}{36}$$
$$\Pr[Z=2] = \frac{9}{36}$$
$$\Pr[Z=3] = \frac{7}{36}$$
$$\Pr[Z=4] = \frac{5}{36}$$
$$\Pr[Z=5] = \frac{3}{36}$$
$$\Pr[Z=6] = \frac{1}{36}$$



Grading: For full credit, a valid plot must be provided, as well as clearly stating the correct probabilities in the distribution of Z.

Problem 2. [Trapping Ants] (27 points)

An ant starts in the lower-left corner of the following grid (i.e., at $S = (0,0)$) and wants to get to the upper-right corner $F = (4,4)$:



The ant moves from one cell to an adjacent cell subject to the following restrictions: the ant only moves up or right (never left, down, or diagonal), and the ant never goes outside the grid. Answer the following questions. Unless specified, you do not need to show your work.

(a) (3 points) Consider sequences of the letters R and U, where R means that the ant moves right and U means that the ant moves up. Give an example of one such sequence that corresponds to a path from S to F. How many paths can the ant take to get from S to F?

> $RRRUUUUR$. $\binom{8}{4}$.

Comment: We need to choose 4 out of the 8 positions in the 8-letter sequence to be R.

Grading: In order to receive full credit, a valid sequence and the correct number of paths must be provided.

(b) (3 points) Now, assume there is a single ant trap, located in the center of the grid, at cell $(2,2)$. How many paths are dangerous, i.e., how many paths from S to F go through the trap?

$\boxed{\binom{4}{2} \times \binom{4}{2}.}$

Comment: There are $\binom{4}{2}$ ways to get from S to $(2,2)$ and $\binom{4}{2}$ ways to get from $(2,2)$ to F.

Grading: In order to receive full credit the correct number of paths must be provided.

Common mistakes: A number of students apparently only computed the number of paths from S to $(2,2)$, namely $\binom{4}{2}$. This is not what the question asked for.

(c) (3 points) More generally, suppose there is a single ant trap, in the cell $(k,4-k)$, where $k \in \{0,1,2,3,4\}$, i.e., in *one* of the cells marked T in the figure below:

T				F
	T			
		T		
			T	
S				T

How many paths are now dangerous? (Your answer should be a function of k.)

$\boxed{\binom{4}{k} \times \binom{4}{4-k} = \binom{4}{k}^2.}$

Grading: Full credit was only given if the correct answer for every k was provided.

Common mistakes: Some students were confused by the term "function," thinking that it refers only to a polynomial. Anything that maps each value in its domain to a unique value in its range is a function. A function does not need to be a polynomial (e.g., $f(x) = x^3$), and when the domain has finite size, it is even sufficient to explicitly list the mapping for each domain element.

(d) (4 points) Now the ant picks a path from S to F uniformly at random from all possible paths and a human picks one of the T cells uniformly at random and puts a trap there. The ant's path and the location of the ant's trap are independently chosen. Using your answers to parts (a) and (c), give an expression for the probability that the ant gets trapped. Leave your answer in terms of a single summation. Show your work.

There are $\binom{8}{4}$ paths and 5 trap locations, so $\binom{8}{4} \times 5$ outcomes, all equally likely. How many outcomes are there where the ant gets trapped? It's the sum, over all k, of the number of paths that go through the trap location $(k,4-k)$. Therefore

$$\Pr[\text{ant gets trapped}] = \frac{\text{number of outcomes where ant is trapped}}{\binom{8}{4} \times 5} = \frac{\sum_{k=0}^{4} \binom{4}{k}^2}{\binom{8}{4} \times 5}$$

Grading: Correct reasoning in terms of the answers to (a) and (c) was necessary in order to receive full credit.

Common mistakes: A common mistake was to forget the factor of 5 corresponding to the number of trap locations.

(e) Now we calculate the probability in part (d) in another way.

(i) (3 points) Let $P_1, P_2, \ldots, P_n$ be the paths that the ant can take from S to F. Consider an arbitrary path P_i. How many times can it pass through a cell marked T? Does your answer depend on the path?

Exactly once. No.

Grading: Both parts of the solution must be provided for full credit, and the answer to the first part must be exactly one. We did not give full credit for answers along the lines of "at most once."

(ii) (4 points) Using your answer to (e)(i) or otherwise, determine the probability that the ant gets trapped conditional on it choosing a particular path P_i. The answer should be a number.

Let Q_i be the event that the ant follows path P_i, R be the event that the ant is trapped. Then $\Pr[R|Q_i] = 1/5$ for all $i \in \{1,2,...,n\}$, where $n = \binom{8}{4}$ is the number of paths.

Grading: Full credit was awarded only for the correct answer.

(iii) (4 points) Using your answer to part (e)(ii), compute the probability that an ant's randomly selected path goes through the randomly selected trap. The answer should be a number and should not involve summations. Show your work.

By the Total Probability Rule,

$$\begin{aligned}
\Pr[R] &= \sum_{i=1}^{n} \Pr[R|Q_i] \times \Pr[Q_i] \\
&= \sum_{i=1}^{n} \frac{1}{5} \times \Pr[Q_i] \\
&= \frac{1}{5} \times \sum_{i=1}^{n} \Pr[Q_i] = \frac{1}{5} \times 1 = \frac{1}{5},
\end{aligned}$$

since the events $Q_1, \ldots, Q_n$ partition the sample space.

Grading: In order to receive full credit, the answer must do the following: (1) demonstrate how to use the Total Probability Rule to compute the probability that the ant is trapped and (2) find that the probability is $\frac{1}{5}$.

Common mistakes: A very common mistake was to merely rephrase part (e)(ii), that the conditional probability $\Pr[R|Q_i] = \frac{1}{5}$ for all i. However, what is required here is a demonstration of how to get from the conditional probabilities $\Pr[R|Q_i]$ to the unconditional probability $\Pr[R]$. This requires the Total Probability Rule.

Another common mistake was to merely evaluate the sum in part (d). Since this question explicitly required the use of the answer in part (e)(ii), we did not award credit for such solutions.

(iv) (3 points) Suppose now the ant does not choose a path uniformly at random but the human still puts the trap uniformly at random on one of the T cells. Does the answer to part (e)(iii) change? Why?

No. The calculation in part (e)(iii) only relied upon the fact that $\sum_{i=1}^{n} \Pr[Q_i] = 1$, which is true for any distribution whatsoever, uniform or not.

Grading: In order to receive full credit, the answer must argue that the computation in part (e)(iii) does not depend on the distribution of the ant's path choices. Again, merely restating part (e)(ii) was insufficient to receive full credit.

Common mistakes: Some students thought that the ant now has the freedom to avoid traps altogether. This is incorrect, since the only thing changed from part (d) is that the ant's path choice is no longer a uniform distribution. The ant still chooses its path independently from the human's choice of trap location.

Problem 3. [Intelligent Guessing] (20 points)

Your friend has a bag with two coins, one of which is a fair coin and the other a trick coin with Heads on both sides. The game is that he picks one of the coins randomly and flips it several times, and you guess which coin he picked based on the outcome of the flips.

(a) (3 points) Your friend picks a coin from the bag uniformly at random and flips it once, resulting in a Heads. Which coin would you guess? What is the probability that you are wrong? Show your work.

The trick coin. Let F denote the event that your friend picks the fair coin, and H the event that we observe a Heads.

$$\Pr[\text{wrong}] = \Pr[F|H] = \frac{\Pr[F \cap H]}{\Pr[H]} = \frac{\frac{1}{2}\times\frac{1}{2}}{\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times 1} = \frac{1}{3}.$$

Grading: For full credit, the answer must do three things: (1) answer "trick coin" to the first question, (2) show a method for calculating the probability $\Pr[F|H]$, and (3) state that the probability you are wrong is $1/3$.

(b) (2 points) Your friend picks a coin from the bag uniformly at random and flips it twice, resulting in Heads once and Tails once. Which coin would you guess? What is the probability that you are wrong? Show your work.

The fair coin. $\Pr[\text{wrong}] = 0$, since there are no outcomes where we are wrong.

Grading: For full credit, the answer must guess "fair coin" and say that the probability you are wrong is 0.

(c) (5 points) Your friend picks a coin from the bag uniformly at random and flips it twice, resulting in two Heads. Which coin would you guess? What is the probability that you are wrong? Show your work.

The trick coin. Let HH denote the event that we observe two Heads.

$$\Pr[\text{wrong}] = \Pr[F|HH] = \frac{\Pr[F \cap HH]}{\Pr[HH]} = \frac{\frac{1}{2}\times\frac{1}{2}^2}{\frac{1}{2}\times\frac{1}{2}^2+\frac{1}{2}\times 1^2} = \frac{1}{5}.$$

Grading: For full credit, the answer must do four things: (1) guess "trick coin", (2) identify that we are computing the probability $\Pr[F|HH]$ (or equivalent), (3) provide a correct and clearly communicated method for computing this probability, (4) find that the probability you are wrong is $1/5$.

Common mistakes: A very common mistake was failing to define the events and probabilities we are trying to calculate (e.g., omitting (2) above). Some people started writing down expressions like $\frac{1/8}{1/8+1/2}$ without identifying what they were computing, or even tried to just write down the answer from scratch. Such solutions did not receive full credit. Some people enumerated outcomes in a sample space, but often without identifying the probability assignment and without showing where the numbers they got came from.

Another common mistake was to get confused about the difference between (i) your friend randomly picks one coin and flips that coin twice vs. (ii) your friend randomly picks a coin, flips it, returns it to the bag, randomly picks a coin (a second time), and flips it. This question was asking about situation (i). In situation (i), $\Pr[HH] = 5/8$, as calculated above; if we were in situation (ii), we would have $\Pr[HH] = (3/4)^2 = 9/16$, which is different. Unfortunately, when this mistake occurred, it also tended to carry over to parts (d) and (e), costing points there as well.

(d) (5 points) Now suppose your friend did not pick uniformly at random from the two coins but instead picks the fair coin with probability p and the trick coin with probability $1-p$. What is the range of values of p for which you will change the guess you made in part (c)? Show your work.

> If I guess the trick coin,
>
> $$\Pr[\text{wrong}] = \Pr[F|HH] = \frac{\Pr[F \cap HH]}{\Pr[HH]} = \frac{p \times \frac{1}{2}^2}{p \times \frac{1}{2}^2 + (1-p) \times 1^2} = \frac{p}{4-3p}.$$
>
> I'll change my guess if this is greater than $\frac{1}{2}$, i.e., if $\frac{p}{4-3p} > \frac{1}{2}$, i.e., if $2p > 4-3p$, i.e., if $5p > 4$, i.e., if $p > \frac{4}{5}$.
> I will change my guess if $\frac{4}{5} < p \leq 1$.

Grading: For full credit, the answer must do four things: (1) guess "trick coin", (2) identify that the relevant condition is probability $\Pr[F|HH] > 1/2$ (or $\Pr[F|HH] > \Pr[\overline{F}|HH]$), (3) compute this probability correctly, (4) find that the range is $p \geq 4/5$.

Common mistakes: Some used the condition $\Pr[F \cap HH] > \Pr[\overline{F} \cap HH]$. This happens to give the right answer (as can be proven by careful examination of Bayes' rule), but without justification for why this condition is the right threshold for when you should change your mind, it did not receive full credit. Others suggested the condition $\Pr[HH|F] > \Pr[HH|\overline{F}]$. This condition does not lead to the correct answer. (For part (c), it happens to be equivalent to the previous condition, because in part (c) both the fair and trick coin were equally likely; but in part (d), it is no longer valid.)

By far the most common mistake was to attempt to start writing down some formulas. Typically, it was unclear where these formulas came from, so these answers did not receive full credit. Also, students who did not set up the problem by identifying the events, probabilities, and condition of interest were more likely to go astray somewhere.

(e) (5 points) Now suppose you are conservative and are not willing to guess unless you have less than a 10% chance of making a mistake. Assuming that your friend picks one of the two coins uniformly at random, how many Heads do you have to observe in a row before you are willing to guess? Show your work.

> Suppose I observe k heads; call this event H^k. If I guess the trick coin,
>
> $$\Pr[\text{wrong}] = \Pr[F|H^k] = \frac{\Pr[F \cap H^k]}{\Pr[H^k]} = \frac{\frac{1}{2} \times \frac{1}{2}^k}{\frac{1}{2} \times \frac{1}{2}^k + \frac{1}{2} \times 1^k} = \frac{1}{1+2^k}.$$
>
> Let's set $\frac{1}{1+2^k} < 0.1$ and solve for k. Equivalently, $1+2^k > 10$, i.e., $2^k > 9$, i.e., $k \geq 4$.
> I have to see 4 Heads before I'm willing to guess; then I guess the trick coin.

Grading: For full credit, the answer must: (1) identify the correct condition explicitly (namely, $\Pr[F|H^k] < 0.1$, or equivalent), (2) compute this conditional probability correctly and clearly, and (3) solve successfully for the number of heads and state that 4 heads are needed.

Common mistakes: A common error was to compute $\Pr[H^k|F]$ (instead of $\Pr[F|H^k]$) and state that we need to have $k = 4$ flips before this probability goes below 0.1. This answer received no credit. Typically, no credit was provided for answers that applied an incorrect method and happened to arrive at 4 heads by coincidence.

Comments: The most common shortcoming in student answers was to try to answer the problem through intuition, without setting up the problem carefully. This doesn't scale: the more complicated the problem, the greater the chances that you go astray. I would recommend many students to take a quick refresher on "the recipe" for solving probability problems, and work on following this recipe on the final exam. Some advice: identify the sample space and probability assignment, if it is not clear (make sure to define any notation you use); define and specify the events of interest (introduce variables to represent the events, if they are not specified in the problem, and provide an English definition of each); state what probabilities you are computing, expressed formally in terms of these events; show steps in the probability computation, applying standard rules of probability.

Another common shortcoming was failing to communicate clearly the method that you are using. When we ask you to "show your work," we are testing not just your ability to compute the final answer, but also your ability to communicate clearly the steps of the computation to others and to ensure that each step is justified. Frequently, I saw written answers that focused on the details (e.g., the algebra of the specific arithmetical computation) without providing the overall structure of the computation (e.g., the part that clarifies what is being computed). Some advice for the final: When we ask you to "show your work," don't think of this as us asking to see the sheet of scratch paper you used as you worked out the final answer; that scratch paper would omit many key points (such as the overall approach, or what you are computing) which are in your head and which you have not written down because you didn't need them for your own self—but which we do want to see. One possible rule of thumb would be to do the calculation for yourself, just to get the final answer, on scratch paper; then identify the overall structure of your computation and write down on your exam paper the full calculation in a logical step-by-step fashion, in a form that will communicate your approach and your calculation clearly to readers in sufficient detail to enable them to verify it for themselves.

Problem 4. [Weather Forecasting] (28 points)

The weather forecast says that each of the next 7 days has a 50% chance of rain and 50% chance of sun. Each day is either rainy or sunny (those are the only two possibilities, and both are equally likely), and the weather on any day is independent of all the other days.

Define the events $R_1, \ldots, R_7$ as follows: R_i is the event that it is rainy on the ith day. Let the random variable X denote the total number of days that it rains, out of the next 7 days.

(a) (1 point) Are the events R_1, R_2 independent? You don't need to justify your answer.

$\boxed{\text{Yes.}}$

(b) (1 point) Are the events $R_1, R_2, \ldots, R_7$ mutually independent? You don't need to justify your answer.

$\boxed{\text{Yes.}}$

(c) (4 points) Calculate the following probabilities. You do not need to show your work.

$\Pr[X=0] = \boxed{\frac{1}{2^7}}$ $\Pr[X=1] = \boxed{\frac{7}{2^7}}$

$\Pr[X=2] = \boxed{\frac{\binom{7}{2}}{2^7}}$ $\Pr[X=3] = \boxed{\frac{\binom{7}{3}}{2^7}}$

$\Pr[X=4] = \boxed{\frac{\binom{7}{4}}{2^7}}$ $\Pr[X=5] = \boxed{\frac{\binom{7}{5}}{2^7}}$

$\Pr[X=6] = \boxed{\frac{7}{2^7}}$ $\Pr[X=7] = \boxed{\frac{1}{2^7}}$

(d) (4 points) Calculate the expectation of X. Show your work. Circle your final answer. Your final answer should be a number (not an unevaluated expression).

$X \sim \text{Binomial}(7, \frac{1}{2})$. Therefore, $\mathbb{E}(X) = 7 \times \frac{1}{2} = \boxed{7/2}$.

Alternative answer: Let $X_i = 1$ if it rains on day i, or 0 otherwise, so $\mathbb{E}(X_i) = \Pr[X_i = 1] = \Pr[R_i] = \frac{1}{2}$. Then $X = X_1 + \cdots + X_7$, so by linearity of expectation, $\mathbb{E}(X) = \mathbb{E}(X_1) + \cdots + \mathbb{E}(X_7) = \frac{1}{2} + \cdots + \frac{1}{2} = \boxed{\frac{7}{2}}$.

Comment: I would not recommend trying to solve this problem by computing the distribution of X and then applying the formula for the expectation. You could do it, but it requires some tedious arithmetic before you get to a number. Why create unnecessary work for yourself, not to mention unnecessary opportunities for mistakes? Work smart, not hard.

Grading: If you used linearity of expectation to solve the problem, you need to define the random variables X_i in order to get full credit.

Common mistakes: Many students were computing expectations of the events R_i instead of the corresponding random variables X_i. There is no such thing as taking expectations of events. Whenever you want to 'count' an event like R_i, you should define the random variable X_i like in the second solution. These X_i are called indicator random variables, since they take value 1 only for points in the event R_i and hence indicate membership in R_i.

The weather service cancels the previous forecast and issues a new weather forecast. The new weather forecast says that there is a 50% chance of rain and a 50% chance of sun on the first day; also, on each subsequent day, there is a $\frac{2}{3}$ chance that the weather is the same as the previous day, and a $\frac{1}{3}$ chance that the weather is the opposite of what it was on the previous day. Each day is either rainy or sunny (those are the only two possibilities).

(e) (4 points) With the new forecast, calculate the following probabilities. You do not need to show your work.

$\Pr[R_1] = \boxed{\frac{1}{2}}$ $\Pr[R_2] = \boxed{\Pr[R_1]\Pr[R_2|R_1] + \Pr[\overline{R_1}]\Pr[R_2|\overline{R_1}] = \frac{1}{2} \times \frac{2}{3} + \frac{1}{2} \times \frac{1}{3} = \frac{1}{2}}$

$\Pr[R_3] = \boxed{\frac{1}{2} \times \frac{2}{3} + \frac{1}{2} \times \frac{1}{3} = \frac{1}{2}}$ $\Pr[R_4] = \boxed{\frac{1}{2}}$

$\Pr[R_5] = \boxed{\frac{1}{2}}$ $\Pr[R_6] = \boxed{\frac{1}{2}}$

$\Pr[R_7] = \boxed{\frac{1}{2}}$

(f) (2 points) With the new weather forecast, are the events R_1, R_2 independent? Why?

$\boxed{\text{No. } \Pr[R_1 \cap R_2] = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3}, \text{ but } \Pr[R_1] \times \Pr[R_2] = \frac{1}{2} \times \frac{1}{2} \neq \frac{1}{3}.}$

Revised 1/3: to fix an error in the calculation of $\Pr[R_1 \cap R_2]$.

Grading: In order to get full credit, you need to give the condition that has to hold for R_1 and R_2 to be independent (like in the solution above) and show why that condition doesn't hold for R_1 and R_2 by explicitly giving the appropriate probabilities, instead of arguing intuitively that there is a dependence between R_1 and R_2.

(g) (2 points) With the new weather forecast, are the events $R_1, R_2, \ldots, R_7$ mutually independent? Why?

$\boxed{\text{No. } R_1, R_2 \text{ are not independent, as shown in part (f).}}$

(h) (5 points) With the new forecast, calculate the following probabilities. You do not need to show your work.

$$\Pr[X = 0] = \boxed{\frac{1}{2} \times \frac{2}{3}^6}$$

$$\Pr[X = 1] = \boxed{2 \times \frac{1}{2} \times \frac{1}{3} \times \frac{2}{3}^5 + 5 \times \frac{1}{2} \times \frac{1}{3}^2 \times \frac{2}{3}^4} \left(= \frac{8}{81}\right)$$

Comment: $X = 0$ only happens if the first day is sunny (probability $\frac{1}{2}$) and every subsequent day is sunny as well (contributes a factor of $\frac{2}{3}$ for each day after the first).

The outcomes that correspond to $X = 1$ are *rssssss*, *srsssss*, *ssrssss*, *sssrsss*, *ssssrss*, *sssssrs*, and *ssssssr* (where *s* denotes a sunny day and *r* a rainy day). The outcomes *rssssss* and *ssssssr* involve a single switch between rainy/sunny, so each has probability $\frac{1}{2} \times \frac{1}{3} \times \frac{2}{3}^5$. (For instance, $\Pr[rssssss] = \frac{1}{2} \times \frac{1}{3} \times \frac{2}{3}^5$ and $\Pr[ssssssr] = \frac{1}{2} \times \frac{2}{3}^5 \times \frac{1}{3}$.) The other five outcomes involve two switches between rainy/sunny, and thus each of them has probability $\frac{1}{2} \times \frac{1}{3}^2 \times \frac{2}{3}^4$.

(i) (4 points) With the new weather forecast, calculate the expectation of X. Show your work. Circle your final answer. Your final answer should be a number (not an unevaluated expression).

$\boxed{\text{Let } X_i = 1 \text{ if it rains on day } i, \text{ or 0 otherwise, so } \mathbb{E}(X_i) = \Pr[X_i = 1] = \Pr[R_i] = \frac{1}{2}. \text{ Then } X = X_1 + \cdots + X_7, \text{ so by linearity of expectation, } \mathbb{E}(X) = \mathbb{E}(X_1) + \cdots + \mathbb{E}(X_7) = \frac{1}{2} + \cdots + \frac{1}{2} = \boxed{\frac{7}{2}}.}$

Comment: You *really* don't want to try to solve this problem by computing the distribution of X and then applying the formula for the expectation. It gets horrifically ugly.

Grading: Again, to get full credit, you need to explicitly define the random variables X_i or refer to part (d) if you did it over there.

Common mistakes: Like in part (d), many students were taking expectations of the events R_i.

(j) (1 point) Is your answer to part (i) larger than, the same as, or smaller than your answer to part (d)? You don't need to justify your answer.

$\boxed{\text{The same.}}$

Comment: The random variable X_i has the same distribution in (d) and (i) and hence the expectation of $X_1 + \ldots + X_7$ is the same in both cases. However, the distribution of $X_1 + \ldots + X_7$ is different in the two cases, because in the first case the events $R_1, \ldots, R_7$ are independent while in the second case they are dependent.