MATH 53 – MIDTERM 1

Each problem counts 20 points.

Problem #1. (a) Compute ∇f for

$$f(x,y) = e^{x^2y+\sin(xy)}$$

(b) Compute ∇g for

$$g(x,y,z) = (x^2+y^3+z^4)^{-1}$$

Problem #2. Find the critical points of the function

$$f(x,y) = x^4 + 2y^2 - 4xy$$

and classify each as a local maximum, local minimum or saddle point.

Problem #3. The position vector **r**(t) of a particle moving in three dimensions satisfies

$$\mathbf{r}' = \mathbf{r} \times \mathbf{a}$$

where **a** is a fixed vector.

Show that either the particle is not moving or else its motion lies within a circle.

(Hint: Show |**r**| and **r · a** are constant.)

Problem #4. Find the area of the region inside the curve

$$r = 4\sin 2\theta$$

and outside the circle

$$r = 2$$

for $0 \le \theta \le \frac{\pi}{2}$.

(Reminders: $\sin\frac{\pi}{6} = \frac{1}{2}, \sin^2 x = \frac{1-\cos 2x}{2}$)

Problem #5. Assume that the two equations

$$f(x,y,z) = 0, g(x,y,z) = 0$$

together implicitly define y as a function of x and z as a function of x.

Find formulas for $y' = \frac{dy}{dx}$ and $z' = \frac{dz}{dx}$ in terms of the partial derivatives of f and g.