

CE100
Midterm Examination
Fall 2010
Friday, October 1, 2010

Name Solutions

Student I.D. _____

This exam is open book and open notes. You will be given fifty (50) minutes to complete two problems. Space is provided on each page for your solution, the back of the pages may also be used. Note that the first problem is worth 40 points and the 2nd is worth 10; allocate your time accordingly.

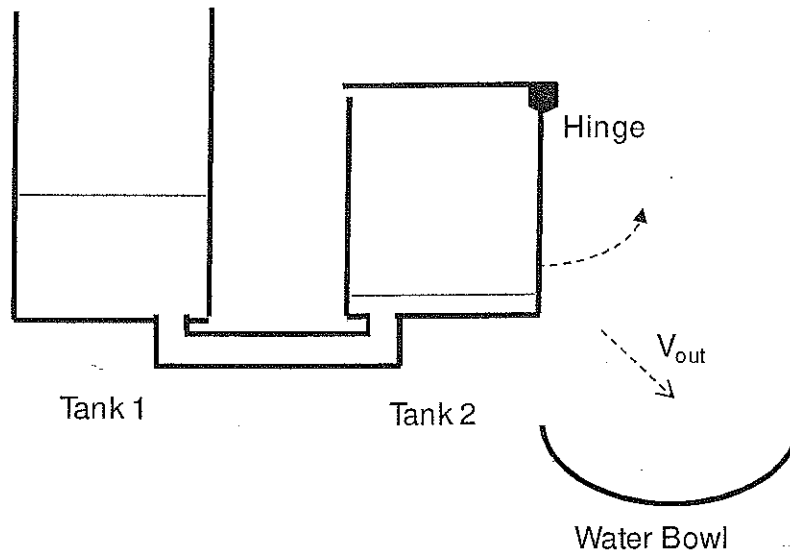
State clearly any assumptions you use in the solutions. Good Luck!

For reference:

Atmospheric Pressure = $p_{\text{atm}} = 100 \text{ kPa}$
Gravitational Acceleration = $g = 9.8 \text{ m/s}^2$

Problem 1 (40 points):

The set of tanks shown in the following diagram is designed to intermittently fill a water bowl for a pet dog during an owner's absence. The system consists of two tanks, connected by a small connecting pipe, and the water bowl. The first tank is maintained at a constant depth, $H_1 = 0.15$ m. The right-hand-wall of the second tank is hinged at the top, with a locking mechanism that can resist a moment of $M_0 = 2.65$ Nm. When a counter-clockwise moment larger than M_0 is applied around the hinge, the wall will open momentarily and allow all water in tank 2 to drain out into the water bowl.



Additional measurements include:

Depth of tank 1 (constant): $H_1 = 0.15$ m

Surface area of tank 1: $A_1 = 0.0625$ m²

Surface area of tank 2: $A_2 = 0.0625$ m²

Cross-sectional area of connecting pipe: $A_{pipe} = .0001$ m²

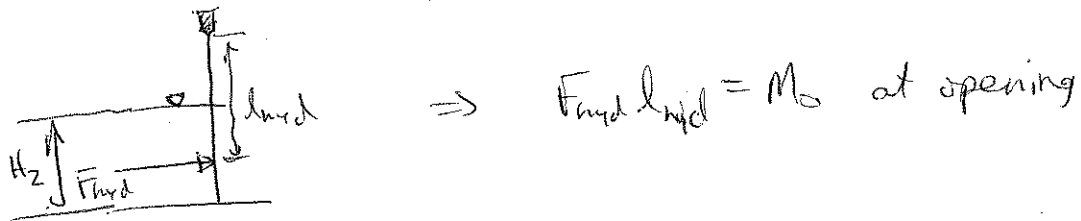
Height of the hinge above bottom of tank 2: $L = 0.25$ m

Width of each tank into the page: $W = 0.25$ m

Assuming that the second tank starts empty and that water is added to the first tank to ensure H_1 is constant:

- Determine the depth of water in tank 2 when the gate opens. An exact solution to the equation is not necessary, an estimate to the nearest 5 cm is sufficient.
- Initially, tank 2 is completely empty. For this condition, determine the velocity of flow going from tank 1 to tank 2.
- Based on your answers to (a) and (b), you can define an estimate for how long it will take for the wall to open by assuming the velocity calculated in (b) is constant as tank 2 fills. In reality, will it take a longer or shorter time (than an estimate based on constant velocity) for the wall to open? Explain your reasoning.

(1a) Require hydrostatic moment around hinge to exceed $2.65 \text{ Nm} = M_0$



$$F_{\text{hyd}} = P_c A = \gamma \frac{H_2}{2} \cdot (H_2 \cdot W) = \frac{1}{2} \gamma H_2^2 W$$

$$l_{\text{hyd}} = L - H_2 + y_R ; \quad y_R = \frac{2}{3} H_2$$

$$\Rightarrow l_{\text{hyd}} = L - \frac{1}{3} H_2$$

$$\Rightarrow F_{\text{hyd}} \cdot l_{\text{hyd}} = \frac{1}{2} (L - \frac{1}{3} H_2) \gamma H_2^2 W = M_0 \text{ for open.}$$

Nearest 0.05 m: $H_2 = 0.1 \text{ m}$

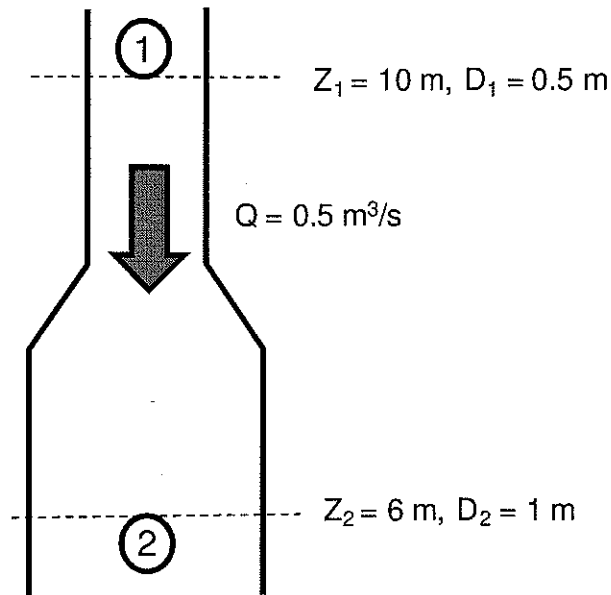
(1b) Bernoulli's from surface of tank 1 to outlet in 2:

$$\left. \begin{aligned} \frac{P_1}{\rho} + z_1 + \frac{V_1^2}{2g} &= \frac{P_2}{\rho} + z_2 + \frac{V_2^2}{2g} \\ \text{0 gage} & \quad \text{0 (large tank assumption)} \\ z_1 - z_2 &= H_1 \end{aligned} \right\} \boxed{V_2 = \sqrt{2gH_1}}$$

(1c) As tank 2 fills, head difference ($z_1 - z_2$) is reduced, which would lead to a decreased velocity of flow between the two tanks \Rightarrow longer time.

Problem 2 (10 points):

A vertical pipe carrying flow $Q = 0.5 \text{ m}^3/\text{s}$ downwards has an expansion from $D_1 = 0.5 \text{ m}$ to $D_2 = 1 \text{ m}$:



Use Euler's equation to determine the pressure difference between points 1 and 2. Could you have used the Bernoulli equation to do this calculation? Why or why not?

Vertical Euler's Eqn:

$$-\frac{d}{dz}(p + \gamma z) = \rho a_z$$

$$\text{Steady flow} \Rightarrow a_z = w \frac{dw}{dz} \quad (u=v=0)$$

$$\Rightarrow a_z = \frac{1}{2} \frac{d}{dz} (w^2)$$

$$\text{Euler's} \Rightarrow -\frac{dp}{dz} = \rho \frac{d}{dz} \left(\frac{1}{2} w^2 \right) + \frac{d}{dz} (\gamma z)$$

$$\Rightarrow \frac{dp}{dz} = -\frac{\rho}{2} \frac{d}{dz} (w^2) - \gamma$$

integrate from ① to ②:

$$\boxed{P_1 - P_2 = -\frac{1}{2} \rho (w_1^2 - w_2^2) - \gamma (z_1 - z_2)}$$

Yes Bernoulli's is ok. ①+② on streamline, flow is steady