UNIVERSITY OF CALIFORNIA
COLLEGE OF ENGINEERING

E7: INTRODUCTION TO COMPUTER PROGRAMMING FOR SCIENTISTS AND ENGINEERS

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Second Midterm Exam—April 14, 2010

[30 points ~ 45 minutes]

Question	Points
Part I	7
Part II	5
Part III	4
Part IV	2
Part V	7
Part VI	5
TOTAL	**30**

Notes:

1. Your exam should have 19 pages. Check this before you begin.
2. You may use a calculator, your notes, and the textbook on this examination as necessary provided that you do not impede those sitting next to you. No electronic devices are permitted.
3. Use a #2 pencil and green scantron sheet to record your answers. Bubble in your solution to each question on the corresponding space on your scantron. There is one correct answer for each question. Multiple bubbles, incomplete bubbles, or stray marks will cause your solution to be marked incorrect.
4. Please write your **name**, **student ID number**, and **discussion section** on your scantron for identification purposes.
5. You may NOT leave the exam room before the exam ends.

Part I:

Questions 1-8: Systems of Linear Equations

1. We want to solve the following equations for x, y, and z using the command

```
>> X = pinv(A)*B
```

Which of the following constitute a valid A, X, and B for this problem formulation?

y = a*x + b
z = c*y + d
y + z = -x

a) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} -b/a \\ -d/c \\ 1 \end{bmatrix}$

b) $A = \begin{bmatrix} -a & 1 & 0 \\ 0 & -c & 1 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} b \\ d \\ 0 \end{bmatrix}$

c) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, X = \begin{bmatrix} b \\ d \\ z \end{bmatrix}, B = \begin{bmatrix} y - a * x \\ z - c * y - 1 \\ -x - y \end{bmatrix}$

d) $A = \begin{bmatrix} 1 & -1/a & 0 \\ 0 & 1 & -1/c \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} b/a \\ d/c \\ 0 \end{bmatrix}$

2. Consider the matrix $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$. If rank(A) is to evaluate to 3, which conditions must be satisfied?

a) and(eq(a,0), and(eq(b,0), eq(c,0)))
b) and(ne(a,0), and(ne(b,0), ne(c,0)))
c) and(ne(a,0), and(eq(b,0), eq(c,0)))
d) none of the above

3. Consider the matrices $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$. How many solutions X are there to the linear matrix equation A*X=B?

a) zero
b) one
c) three
d) infinity

4. Consider matrices A and B where size(A) = =[m, n] and size(B)= =[m, 1]. Which of the following commands ALWAYS evaluates to true?

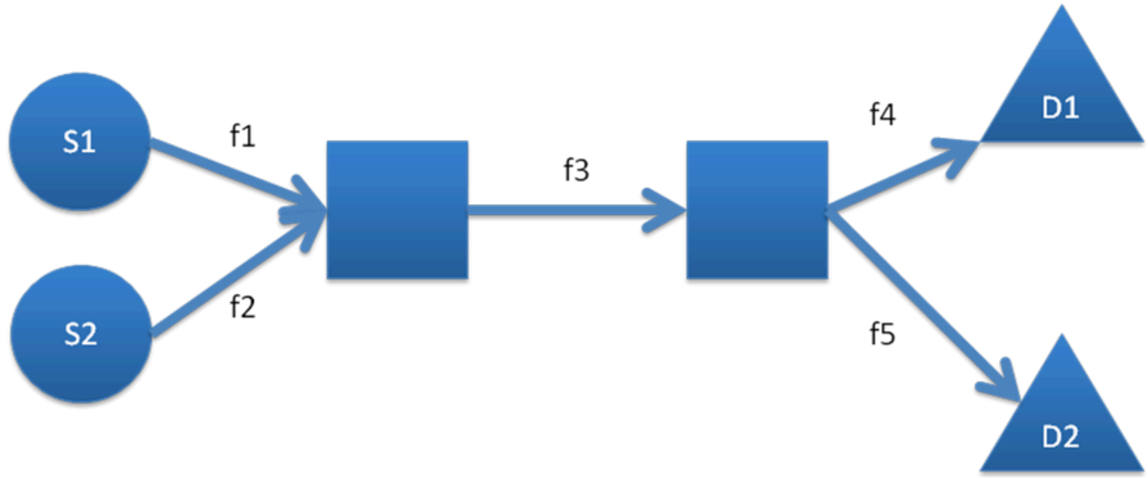
a) `>> X=inv(A)*B; isequal(B,A*X)`
b) `>> X=pinv(A)*B; isequal(B,A*X)`
c) `>> X=A\B; isequal(B,A*X)`
d) none of the above

5. Which of the following commands computes the area of the parallelogram with vertices at
(-2,-1), (1,-1), (1,2), (4,2)? Assume the following variables already exist in the Matlab workspace:

V1 = [-2 -1]
V2 = [1 -1]
V3 = [1 2]
V4 = [4 2]

a) `>> area = abs(sum (V1.*V4))`
b) `>> area = abs(det([V1;V2]))`
c) `>> area = abs(det([V3-V1;V4-V2]))`
d) `>> area = abs(det([V2-V1;V3-V1]))`
e) `>> area = abs(sum(V2.*V3))`

Use this chart for problems 6-7:



You are an engineer operating a water network. The pipes are as shown above. The square boxes denote pipe junctions. S1 and S2 are water sources and D1 and D2 are water consumptions areas. Flows f1 and f2 can be controlled using pipe valves. Your job is to pick the flows in each pipe so as to deliver as much water as possible given the following constraints:

- The sum of the inflows must equal the sum of the outflows at each junction.
- The water pressures at the two sources are such that f1 = 0.5*f2.
- The capacity of flow f3 is 6000 gallons/minute.

6. Scenario 1: The destinations will pay you for as much water as you can give them, and you want to maximize the revenue of the network operator. Which of the following statements is true:

a) There are too many constraints. They cannot all be satisfied. The best flows should be found by a method like least squares that will violate at least some of the constraints.
b) The best flows f1 and f2 can be found by a matlab program of the form inv(A)*b for some matrices A and b.
c) The best flows f1 and f2 can be found by a matlab program of the form pinv(A)*b. The program inv(A)*b will not work for any A and b.
d) None of the above.

7. Scenario 2: Destination D1 cannot absorb more than 2000 gallons/minute. Destination D2 cannot absorb more than 12000 gallons/minute. Which of the following matrix formulations should be solved to determine the best flows?

a)

$$\begin{bmatrix} 1 & -0.5 & 0 & 0 & 0 \\ 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f1 \\ f2 \\ f3 \\ f4 \\ f5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 6000 \\ 2000 \\ 12000 \end{bmatrix}$$

b)

$$\begin{bmatrix} 1 & -0.5 & 0 & 0 & 0 \\ 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} f1 \\ f2 \\ f3 \\ f4 \\ f5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 6000 \\ 2000 \end{bmatrix}$$

c)

$$\begin{bmatrix} 1 & -0.5 & 0 & 0 & 0 \\ 1 & 1 & -1 & 0 & 0 \\ -1 & -1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} f1 \\ f2 \\ f3 \\ f4 \\ f5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 6000 \\ 2000 \end{bmatrix}$$

d)

$$\begin{bmatrix} 1 & -0.5 & 0 & 0 & 0 \\ 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} f1 \\ f2 \\ f3 \\ f4 \\ f5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2000 \end{bmatrix}$$

Part II:
Questions 8-12: Computer Representation of Numbers

The IEEE 754 standard specifies a single precision number in the general form:

$$(-1)^{Sign} \times 2^{Exponent} \times Fraction$$

- *Sign*: 1 bit
- *Exponent*: 8 bits biased with +127 (hint: 127 in binary is 0111 1111)
- *Fraction*: 23 bits with the leading number of fraction being always 1 and thus not explicitly stored.

8. Which of the following represents the ***Exponent*** (in total 8 digits) of the decimal number $(12)_{10}$?
   a) 00000010
   b) 10000011
   c) 10000100
   d) 10000010

9. Which of the following represents the ***Fraction*** (in total 23 digits) of the decimal number $(12)_{10}$?
   a) 0100 0000 0000 0000 0000 000
   b) 1000 0000 0000 0000 0000 000
   c) 1100 0000 0000 0000 0000 000
   d) 0010 0000 0000 0000 0000 000

10. Which of the following decimal number has the IEEE754 representation
1 1000 0001 1100 0000 0000 0000 0000 000?
   a) -8
   b) -7
   c) 7
   d) 8

11. Consider the following numbers, shown below in single precision format:

A = 0 0111 1111 0000 0000 0000 0000 0000 001
B = 0 0111 1111 0000 0000 0000 0000 0000 000

What is the result of the subtraction A – B? Express your answer in base 10.

   a) 10^{-23}
   b) 2^{-23}
   c) 2^{104}
   d) None of the above

12. What is the result of y after the following commands are entered?

```
>> x = single(2^100)
>> y = log2(x + 8 - x)
```

a) –Inf
b) 0
c) 3
d) 8

Part III:

Question 13-16: Regression

13. Give a set of n distinct data points (x_i, y_i), let

$$X = \begin{bmatrix} x_1^m & \cdots & x_1^2 & x_1 & 1 \\ x_2^m & & x_2^2 & x_2 & 1 \\ \vdots & \iddots & & & \\ x_n^m & \cdots & x_n^2 & x_n & 1 \end{bmatrix} \quad \text{and} \quad Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_i \\ \vdots \\ y_n \end{bmatrix}$$

(i.e. X is a matrix with n rows and m+1 columns where the k^{th} column is the x values in the data set raised to the $(m+1-k)^{th}$ power).

Select the Matlab command or commands that will give you the coefficients of the least squares regression m^{th} order polynomial.

(i) `>> pinv(X)*Y`
(ii) `>> inv(X'*X)*X'*Y`
(iii) `>> X\Y`

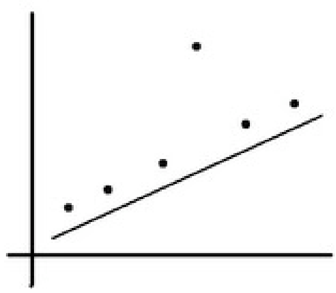
a) i only
b) ii only
c) iii only
d) i, ii, and iii
e) none of the above

14. If there are n = 20 data points , what is the *lowest* number p such that a least squares regression using an p-order polynomial model will be *guaranteed* to give you a sum of squared error of 0.

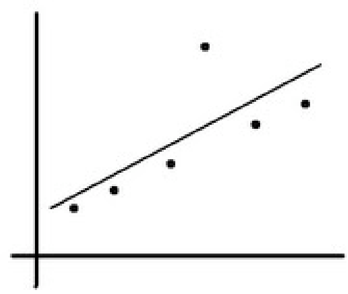
a) 0
b) 1
c) 19
d) 20
e) 21

15. Each figure below shows the data and a possible regression line. The data are the same for each plot. Pick the figure that correctly identifies the least squares fit

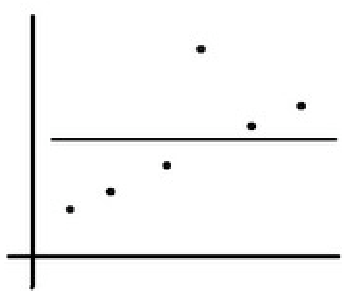
a)



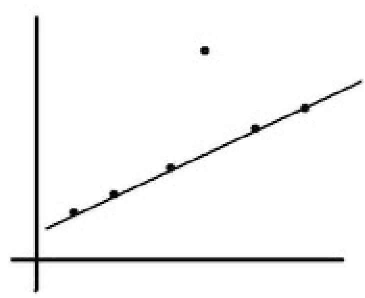
d)



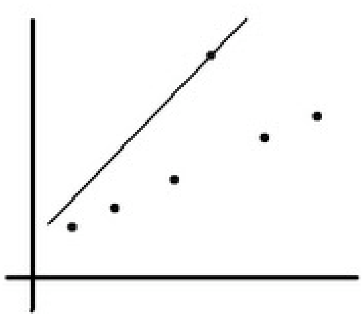
b)



e)



c)



16. Assume you have the following data set:

x_i	y_i
1	8
2	3
3	5
4	4
5	0

Suppose you want to use least squares regression to find the constant (i.e. 0^{th} order polynomial) that best fits the data. If the constant is called k, what will k be for the data set given above?

a) 0
b) 1
c) 2
d) 3
e) 4

Part IV:

Questions 17-18: Interpolation

The following data set applies for question 17:

t = [1 3 8 10 11 16]
y = [24 10 -0.5 5.3 9.0 8.5]

17. What does yy = interp1(t, y, [2, 4:7, 9, 12:15]) do?

a) assigns an array of interpolated values for the times [2, 4:7, 9, 12:15] to yy
b) sets yy = y because interpolation was infeasible
c) produces a Matlab error because the interpolation was infeasible
d) assigns the empty array to yy because the interpolation was infeasible

18. Suppose you have 10 data points and you are fitting a cubic spline interpolation. What information is required to fit the third spline?

a) The slopes of the 2^{nd} and 4^{th} splines
b) The values of the 3^{rd} and 4^{th} data points
c) The values of the 3^{rd}, 4^{th}, and 5^{th} data points
d) The slope of the 2^{nd} spline, and the values of the 3^{rd}, 4^{th}, and 5^{th} data points.

Part V:
Questions 19-25: Root Finding

19. Which of the following statements regarding a bisection method of root finding is true?

> a) For each iteration of a properly implemented bisection method, an upper bound on error is always known
> b) A lower bound and upper bound are optional inputs to a bisection method
> c) A bisection method can only find the roots of polynomial equations
> d) A bisection method will always converge faster than a Newton-Raphson method for the same inputs

For problems 20-22, consider the following program bisect.m.
Note: the function sign(a) in Matlab returns 1 if a is positive, -1 if a is negative, and 0 if a = 0. Hint: You can assume that this code was written by a good programmer and there are no mistakes or tricks in it. **Note that while you should be able to extract the general idea of what this code does, it may not be necessary to understand every line to answer the questions – use your time wisely!!**

```
 function [root,xmiddle] = bisect(fun, xlower, xupper, tol, maxiter)

 keepgoing = true;
 i = 0;
 while keepgoing
     i = i+1;
    xmiddle(i) = (xlower+xupper)/2;
     sign_middle = sign(fun(xmiddle(i)));
     if sign_middle ~= sign(fun(xupper))
         xlower = xmiddle(i);
     else
         xupper = xmiddle(i);
     end
     if i > 1  %implement stopping conditions
         if abs((xmiddle(i)-xmiddle(i-1))/xmiddle(i)) < tol
             keepgoing = false;
         elseif i > maxiter
             keepgoing = false;
         end
     end
 end
 root = xmiddle(i);
end
```

For problems 20-22, consider fun = @(x)x^2-x-2. The roots of this function are x = 2 and x = -1

20. Which of the following commands will return

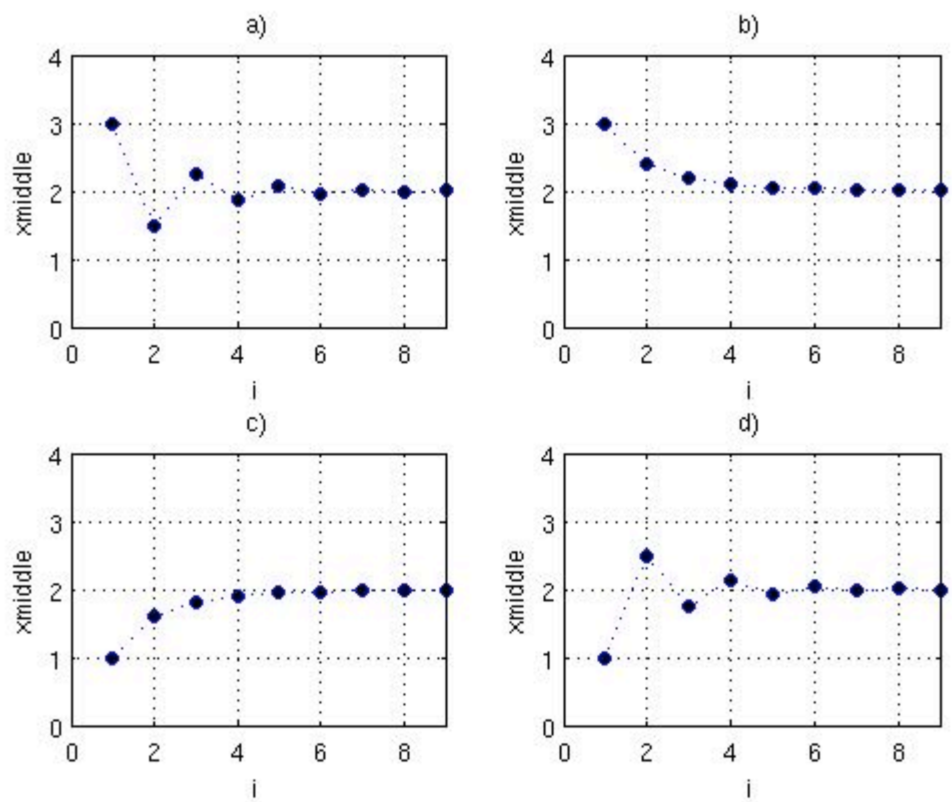
```
root =
    2.0000
```

a) `[root,xmiddle]=bisect(fun,0,5,1E-5,1000); root`
b) `[root,xmiddle]=bisect(fun,-2,1,1E-5,1000); root`
c) `[root,xmiddle]=bisect(fun,-2,0,1E-5,1000); root`
d) None of the above

21. The following command is executed:

```
[root,xmiddle]=bisect(fun,0,6,1E-2,1000);
```

Which of the following graphs of xmiddle versus index is correct?



22. Note that bisect.m is not robust in that it does not include any checking of inputs – whether they are valid, well-chosen, etc. What will the output of the following command be?

```
[root,xmiddle]=bisect(fun,0,1,1E-6,1000); root
```

a)
```
root =
     2.0000
```
b)
```
root =
    -1.0000
```
c) The code will return an incorrect root

d) The code will abort due to an error

23. Which of the following is true regarding the Newton-Raphson method of root finding?

a) The Newton-Raphson method will always converge on a root if a root exists and the function's derivative is continuous everywhere

b) An upper and lower bound are necessary inputs to a Newton-Raphson method

c) At every iteration, the Newton-Raphson method requires that the derivative of the input function be known

d) both (a) and (c) are true

For problems 24 and 25, consider the following program newton.m, a simple implementation of the Newton-Raphson method. **Note that while you should be able to extract the general idea of what this code does, it may not be necessary to understand every line to answer the questions – use your time wisely!!**

```
function [root,xguess] = newton(fun, dfun, x0, tol, maxiter)
keepgoing = true;
i = 1;
xguess(i) = x0;
while keepgoing
    i = i+1;
   xguess(i) = xguess(i-1) - fun(xguess(i-1)) / dfun(xguess(i-1));
    if i > 1   %implement stopping conditions
        if abs((xguess(i) - xguess(i-1)) / xguess(i)) < tol
            keepgoing = false;
        elseif i > maxiter
            keepgoing = false;
        end
    end
end
root = xguess(i);
end
```

Consider that fun = @(x)sin(x) and dfun = @(x)cos(x)

24. What would displayed on the workspace after the following command?
`[root,xguess] = newton(fun,dfun,0.75*pi,1E-6,1000);  root`

a) `root =`
`0`

b) `root =`
`1.5708`

c) `root =`
`3.1416`

d) `root =`
`6.2832`

25. What would be displayed on the workspace after the following command?
`[root,xguess] = newton(fun,dfun,3*pi/2,1E-6,1000);  root`

a) `root =`
`3.1416`

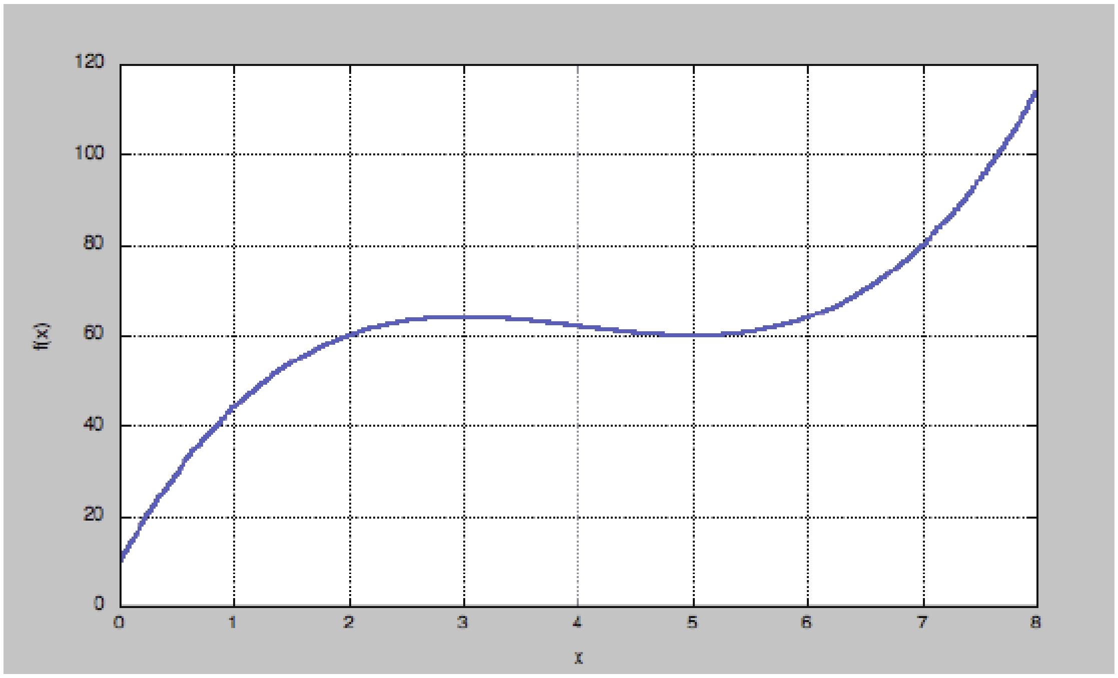
b) `root =`
`4.7124`

c) `root =`
`6.2832`

d) none of the above

Part VI:

Questions 26-30: Numerical Integration and Differentiation



26. For the f(x) described by the graph above, which of the following are true? (*Note that rectangular blocks used for numerical integration are assumed to by calculated via the midpoint rule.*)

a) Calculating $\int_0^4 f(x)dx$ numerically using trapezoidal integration will always lead to an overestimate of the true (analytical) value.

b) Calculating $\int_5^8 f(x)dx$ numerically using trapezoidal integration will always lead to an overestimate of the true (analytical) value.

c) If you use numerical integration with 4 equally spaced rectangular subintervals to calculate $\int_0^4 f(x)dx$, you will get approximately (30+52+63)=145

d) If you use numerical integration with 4 equally-spaced rectangular subintervals to calculate $\int_0^8 f(x)dx$, you will get approximately (45+65+60+80)=250

27. What will be displayed after the following commands are entered in the command window?

```
>> x= 0:.01:5;
>> y=x.^2;
>> z1= trapz(x,y);
>> z2=.01*trapz(y);
>> compare = (z1==z2);
>> compare
```

Hint: you may find this excerpt from the trapz help to be useful

Z = TRAPZ(Y) computes an approximation of the integral of Y via the trapezoidal method (with unit spacing). To compute the integral for spacing different from one, multiply Z by the spacing increment.

Z = TRAPZ(X,Y) computes the integral of Y with respect to X using the trapezoidal method. X and Y must be vectors of the same length, or X must be a column vector and Y an array whose first non-singleton dimension is length(X). TRAPZ operates along this dimension.

a) `compare =`
```
      0
```
b) `compare =`
```
      1
```
c) `compare =`
```
      0.0100
```
d) These commands will produce a MATLAB error

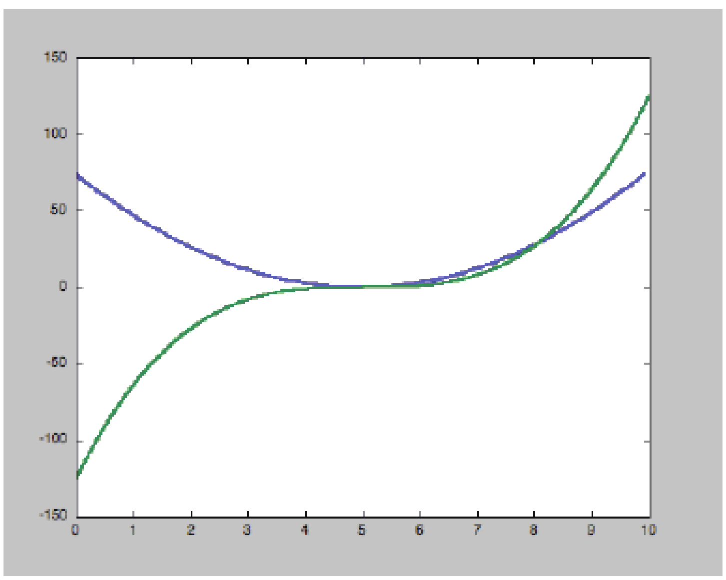
The following code implements a forward difference algorithm:

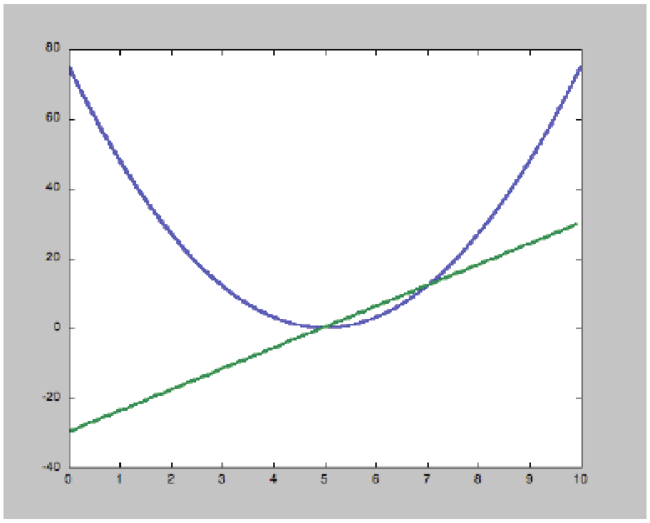
```
function [x, der] = myForwardDiff(myFun,a,b,N)
x = linspace(a,b,N);
dx = (b-a)/(N-1);
der = [];

for i = 1:length(x)-1
der = [der, (myFun(x(i+1)) - myFun(x(i)))/dx];
end

x = x(1:end-1);
end
```

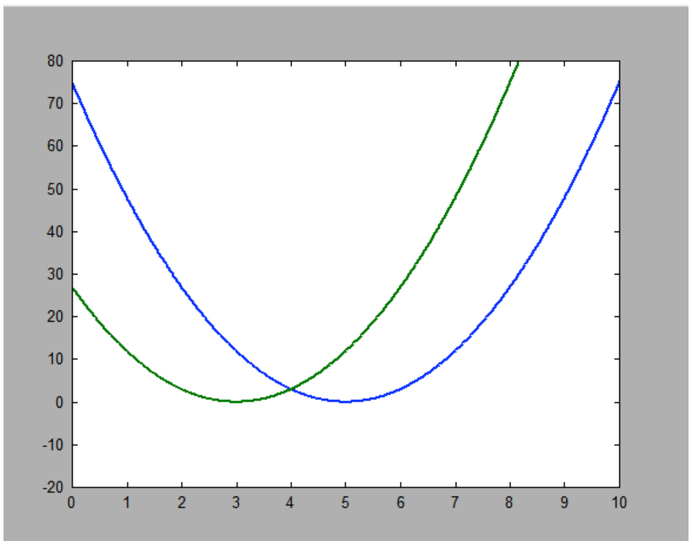
28. What output would be produced by the following commands?

```
>> myFun=@(x) 3*(x-5).^2;
>> N = 100;
>> [xd, der] = myForwardDiff(myFun,0,10,N);
>> x=0:.01:10; y=myFun(x);
>> plot(x,y,xd,der)
```

(a)



(b) 

(c) 

(d) This code would give me a MATLAB error.

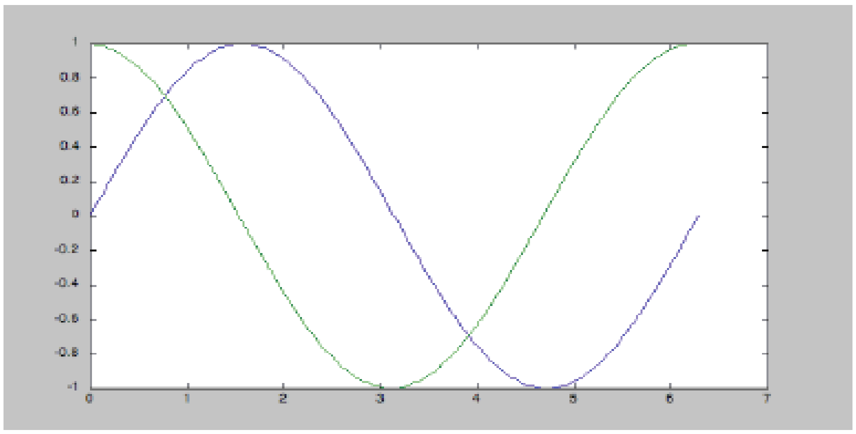
29. After the commands in problem 28 are run, which one of the following conditions will evaluate to true?

a) `ne(myFun(5),0)`
b) `gt(xd(1),0)`
c) `length(xd) == N-1`
d) `length(xd) == N`

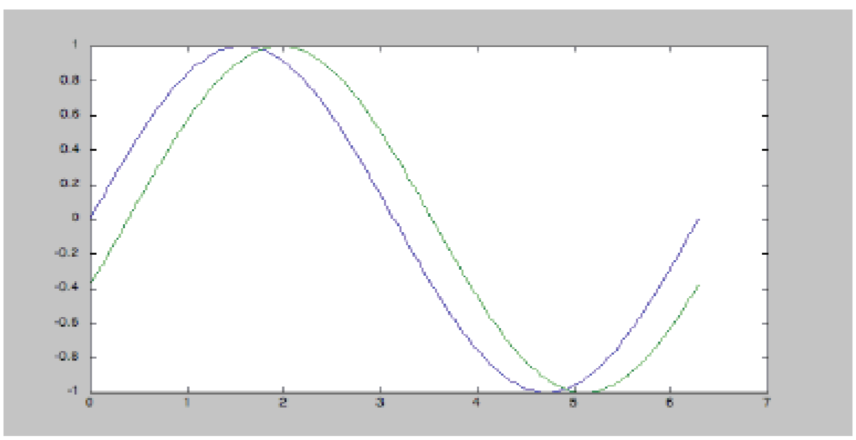
30. What output would be produced by the following commands?

```
>> myFun=@(x) sin(x);
>> [xd, der] = myForwardDiff(myFun,0,2*pi,3);
>> x=0:pi/100:2*pi; y=feval(myFun, x);
>> plot(x,y, xd, der)
```

(a)

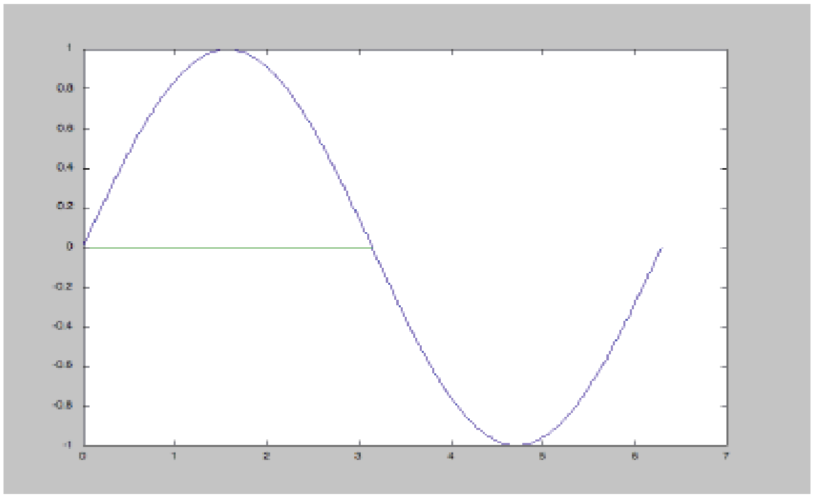


(b)



See next page for options c and d

(c)



(d) none of the above