EECS 20N: Structure and Interpretation of Signals and Systems MIDTERM 1
Department of Electrical Engineering and Computer Sciences 21 September 2010
UNIVERSITY OF CALIFORNIA BERKELEY

LAST Name ______________ FIRST Name ______________

Lab Time ______________

- **(10 Points)** Print your name and lab time in legible, block lettering above AND on the last page where the grading table appears.
- This exam should take up to 70 minutes to complete. You will be given at least 70 minutes, up to a maximum of 80 minutes, to work on the exam.
- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except one double-sided $8.5" \times 11"$ sheet of handwritten notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- **The exam printout consists of pages numbered 1 through 8.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the eight numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this exam.

MT1.1 (45 Points) The instantaneous position of a particle on the complex plane is described as follows:

$$\forall t \in \mathbb{R}, \quad z(t) = \sin(t)\, e^{i\,t}.$$

(a) (10 Points) Determine reasonably simple expressions for the instantaneous Cartesian (rectangular) coordinates of the particle; that is determine $x(t)$ and $y(t)$, where $z(t) = x(t) + i\, y(t)$.

(b) (15 Points) Provide a well-labeled plot of the *trajectory* of the particle on the complex plane. That is, indicate the path and the direction of the particle's motion. To receive credit, you must provide a succinct, but clear and convincing explanation of your work. Please note that we are *not* asking you to plot $x(t)$ and $y(t)$.

(c) (10 Points) Show that the particle's position exhibits periodic behavior, and determine its fundamental period p and fundamental frequency ω_0.

(d) (10 Points) Provide well-labeled plots of $|z(t)|$ and $\angle z(t)$.

MT1.2 (30 Points) Consider two complex numbers x and y.

(a) (7 Points) Prove that $(x + y)^* = x^* + y^*$; that is, the complex conjugate of the sum of two numbers is the sum of their individual complex conjugates.

(b) (7 Points) Prove that $(xy)^* = x^* y^*$; that is, the complex conjugate of the product of two numbers is the product of their individual complex conjugates.

(c) (16 Points) Consider a cubic polynomial

$$Q(z) = \sum_{k=0}^{3} a_k \, z^k = a_0 + a_1 \, z + a_2 \, z^2 + a_3 \, z^3,$$

where every coefficient a_k is real.

(i) (8 Points) Prove that if z is a complex root of the polynomial (i.e., $Q(z) = 0$, and $z \notin \mathbb{R}$), then so is z^*.

(ii) (8 Points) Prove that $Q(z)$ has at least one real root.

MT1.3 (30 Points) For $\Delta > 0$, let $v_\Delta(t)$ be defined by

$$v_\Delta(t) = \begin{cases} 0 & \text{if } t < -\Delta \\ \frac{1}{\Delta} & \text{if } -\Delta \leq t \leq -\frac{\Delta}{2} \\ 0 & \text{if } -\frac{\Delta}{2} < t < \frac{\Delta}{2} \\ \frac{1}{\Delta} & \text{if } \frac{\Delta}{2} \leq t \leq \Delta \\ 0 & \text{if } \Delta < t. \end{cases}$$

(a) (7 Points) Provide a well-labeled plot of v_Δ.

(b) (7 Points) Let f be a function that is continuous at 0. Determine

$$\lim_{\Delta \to 0} \int_{-\infty}^{\infty} f(t)\, v_\Delta(t) dt\,.$$

(c) (6 Points) From the preceding part, how should we represent

$$\lim_{\Delta \to 0} v_{\Delta}(t) \ ,$$

from the point of view of how it behaves inside integrals?

(d) (10 Points) Let f and g be two functions, each of which is continuous at $t = 0$ and at $t = 1$. Determine

$$\int_{-\infty}^{+\infty} \left[\delta(2t) + \delta(t-1)\right] f(t)g(t)dt,$$

where δ denotes the Dirac delta function.

LAST Name ______________________ FIRST Name ______________________

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Problem	Points	Your Score
Name	10	
1	45	
2	30	
3	30	
Total	**115**	

You may or may not find the following information useful:

$$
\begin{aligned}
\cos(\alpha+\beta) &= \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta. \\
\sin(\alpha+\beta) &= \sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta. \\
\cos\alpha + \cos\beta &= 2\cos\left(\frac{\alpha+\beta}{2}\right)\cdot\cos\left(\frac{\alpha-\beta}{2}\right). \\
\sin\alpha + \sin\beta &= 2\sin\left(\frac{\alpha+\beta}{2}\right)\cdot\cos\left(\frac{\alpha-\beta}{2}\right). \\
\delta(at) &= \frac{1}{|a|}\delta(t).
\end{aligned}
$$