UCB Math 110, Fall 2010: Midterm 2

Prof. Persson, November 8, 2010

Name: ______________________________

SID: ______________________________

Section: Circle your discussion section below:

Sec	Time	Room	GSI
01	Wed 8am - 9am	87 Evans	D. Penneys
02	Wed 9am - 10am	2032 Valley LSB	C. Mitchell
03	Wed 10am - 11am	B51 Hildebrand	D. Beraldo
04	Wed 11am - 12pm	B51 Hildebrand	D. Beraldo
05	Wed 12pm - 1pm	75 Evans	C. Mitchell
07	Wed 2pm - 3pm	87 Evans	C. Mitchell
08	Wed 9am - 10am	3113 Etcheverry	I. Ventura
09	Wed 2pm - 3pm	3 Evans	D. Penneys
10	Wed 12pm - 1pm	310 Hearst	I. Ventura

Other/none, explain: ______________________________

Grading

1	/ 18
2	/ 6
3	/ 6
4	/ 10
	/ 40

Instructions:

- One double-sided sheet of notes, no books, no calculators.
- Exam time 50 minutes, do all of the problems.
- You must justify your answers for full credit.
- Write your answers in the space below each problem.
- If you need more space, use reverse side or scratch pages. Indicate clearly where to find your answers.

1. (6 problems, 3 points each) Label the following statements as TRUE or FALSE, giving a short explanation (e.g. a proof or a counterexample).

a) Let $A, B \in \mathsf{M}_{5\times 5}(R)$ such that $AB = -BA$. Then either A or B is non-invertible.

TRUE FALSE (circle one)

b) Every matrix $A \in \mathsf{M}_{5\times 5}(R)$ has an eigenvector in R^5.

TRUE FALSE (circle one)

c) Let $A, B \in \mathsf{M}_{n\times n}(F)$, and suppose A is similar to B. Then A^k is similar to B^k for any positive integer k.

TRUE FALSE (circle one)

1. (cont'd)

d) If 0 is the only eigenvalue of a linear operator T, then $\mathsf{T} = 0$.

TRUE FALSE (circle one)

e) If a matrix $A \in \mathsf{M}_{n\times n}(F)$ can be transformed into a diagonal matrix by a sequence of elementary row operations of type 3, then A is diagonalizable.

TRUE FALSE (circle one)

f) Let V be a finite dimensional vector space and γ be a basis for V^*. Then there exists a basis β for V such that $\beta^* = \gamma$.

TRUE FALSE (circle one)

2. (6 points) Find bases for the null space $\mathsf{N}(\mathsf{L}_A)$ and for the range $\mathsf{R}(\mathsf{L}_A)$ where

$$A = \begin{pmatrix} 1 & -1 & 1 & -1 \\ 4 & -4 & 5 & -2 \\ 2 & -2 & -1 & -8 \end{pmatrix}.$$

3. (6 points) Let $\mathsf{V} = \mathsf{R}^2$ and define $\mathsf{f}, \mathsf{g} \in \mathsf{V}^*$ as follows:

$$\mathsf{f}(x, y) = x + y, \quad \mathsf{g}(x, y) = x - 2y.$$

Find a basis β for V such that its dual basis $\beta^* = (\mathsf{f}, \mathsf{g})$.

4. (10 points) Consider the linear operator T on $\mathsf{P}_3(R)$ defined by

$$\mathsf{T}(p(x)) = (x^2+1)p''(x).$$

Determine if T is diagonalizable, and if so, find a basis β for $\mathsf{P}_3(R)$ such that $[\mathsf{T}]_\beta$ is a diagonal matrix.