UCB Math 110, Fall 2010: Midterm 1

Prof. Persson, October 4, 2010

Name: ______________________________

SID: ______________________________

Section: Circle your discussion section below:

Sec	Time	Room	GSI
01	Wed 8am - 9am	87 Evans	D. Penneys
02	Wed 9am - 10am	2032 Valley LSB	C. Mitchell
03	Wed 10am - 11am	B51 Hildebrand	D. Beraldo
04	Wed 11am - 12pm	B51 Hildebrand	D. Beraldo
05	Wed 12pm - 1pm	75 Evans	C. Mitchell
07	Wed 2pm - 3pm	87 Evans	C. Mitchell
08	Wed 9am - 10am	3113 Etcheverry	I. Ventura
09	Wed 2pm - 3pm	3 Evans	D. Penneys
10	Wed 12pm - 1pm	310 Hearst	I. Ventura

Other/none, explain: ______________________________

Grading

1	/ 18
2	/ 12
3	/ 10
	/ 40

Instructions:

- One double-sided sheet of notes, no books, no calculators.
- Exam time 50 minutes, do all of the problems.
- You must justify your answers for full credit.
- Write your answers in the space below each problem.
- If you need more space, use reverse side or scratch pages.
  Indicate clearly where to find your answers.

1. (6 problems, 3 points each) Label the following statements as TRUE or FALSE, giving a short explanation (e.g. a proof or a counterexample).

a) Let $\mathsf{V} = R^n$ and $\mathsf{W} = R$. Then $\mathcal{L}(\mathsf{V}, \mathsf{W})$ is isomorphic to $\mathsf{P}_n(R)$.

b) Let V be a vector space and $\mathsf{T}, \mathsf{U} : \mathsf{V} \to \mathsf{V}$ be two linear operators. Then $\mathsf{N}(\mathsf{U}) \subseteq \mathsf{N}(\mathsf{TU})$.

c) Let V be a vector space and $\mathsf{T}, \mathsf{U} : \mathsf{V} \to \mathsf{V}$ be two linear operators. Then $\mathsf{R}(\mathsf{U}) \subseteq \mathsf{R}(\mathsf{UT})$.

1. (cont'd)

d) The set $S = \{p \in \mathsf{P}(F) : p'(0) = p(0)\}$ is a subspace of $\mathsf{P}(F)$.

e) Let $\mathsf{T} : R^2 \to R^2$ be a linear transformation. Then $R^2 = \mathsf{N}(\mathsf{T}) \oplus \mathsf{R}(\mathsf{T})$.

f) Let W_1 and W_2 be 3-dimensional subspaces of R^5. Then W_1 and W_2 must have a common nonzero vector.

2. (12 points) Let $\mathsf{T} : \mathsf{M}_{2\times 2}(R) \to \mathsf{M}_{2\times 2}(R)$ be defined by

$$\mathsf{T}(A) = BA - A^t \qquad \text{where } B = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}.$$

a) Prove that T is a linear transformation.

b) Find bases for $\mathsf{N}(\mathsf{T})$ and $\mathsf{R}(\mathsf{T})$.

3. (10 points) Let V be a vector space, and $\mathsf{T} : \mathsf{V} \to \mathsf{V}$ a linear operator. Suppose $x \in \mathsf{V}$ is such that $\mathsf{T}^m(x) = \mathit{0}$ but $\mathsf{T}^{m-1}(x) \neq \mathit{0}$ for some positive integer m. Show that $\{x, \mathsf{T}(x), \mathsf{T}^2(x), \ldots, \mathsf{T}^{m-1}(x)\}$ is linearly independent.