NE 180
Midterm II
Fall Semester 2009
Solutions

November 3, 2010

1. a.

$$T_i(0.9\ a) = 15 \times 10^3 \left(1 - (0.9)^2)\right)^{0.3} = \ 9114.17\text{eV}$$

$$n_i(0.9\ a) = 1.0 \times 10^{20}\ \text{m}^{-3} = 1.0 \times 10^{14}\ \text{cm}^{-3}$$

$$n\tau_i = \frac{2.09 \cdot 10^7 (m_i/m_p)^{1/2} T_i^{3/2}}{\ln \Lambda} = 1.64 \times 10^{12}\ \text{cm}^{-3}\text{s}$$

$$\boxed{\tau_i = 0.016\ \text{s}}$$

$$\omega_{ci} = \frac{qB}{m_i} = 1.53 \times 10^8 \text{s}^{-1}$$

$$\boxed{\omega_{ci}\tau_i = 2.5187 \times 10^6}$$

$$\boxed{\kappa_\perp^i = 2.0 \frac{nT_i\tau_i}{m_i(\omega_{ci}\tau_i)^2} = 1.8093 \times 10^{17} \text{m}^{-1}\ \text{s}^{-1}}$$

1. b.

$$\frac{dT}{dr} = (0.3)\frac{2r}{a^2}\left(1 - \left(\frac{r}{a}\right)^2\right)^{-0.7} = -12951 \ \ \text{eV m}^{-1}$$

$$q'' = -\kappa_\perp \frac{dT}{dr} = \boxed{374.936 \text{ Watt m}^{-2}}$$

$$\text{Area} = (2\pi R)(2\pi a) = 426.367 \text{ m}^2$$

$$\text{Total Ion Heat Flux} = \text{Area} \times q'' = \boxed{159.86 \text{ kW}}$$

1. c.

$$q = 1/(1 - (2/3)(r/a)^2) = 2.17391$$

$$\text{Neoclassical Factor} = Q_{neo} = 2q^2\epsilon^{-3/2} = 57.5218$$

Since $q'' = -Q_{neo}\kappa_\perp \frac{dT(r)}{dr}$, we have

So neoclassical heat flux is

$$-Q_{neo}\kappa_\perp \frac{dT(r)}{dr} = \boxed{9.1945 \text{ MW}}$$

1. d.

$$Q_{ie} = \frac{3m_e}{m_i}\frac{n(T_i - T_e)}{\tau_e}$$

At center, $T_i - T_e = 15 - 10 = 5.0$ keV.

$\tau_e = 2.0 \times 10^{10} T_e^{3/2}/n_e$ with T_e in eV and MKS density. Then $\tau_e = 200.0\ \mu$s. and

$$Q_{ie} = \boxed{266.411 \text{ kW m}^{-3}}$$

1. e.

$$\eta = 2.8 \cdot 10^{-8} Z T_{e(keV)}^{-3/2}$$

$$= 2.24 \times 10^{-10}\ \Omega\,\text{m}$$

$$J_\phi(0) = \frac{2\,B_\phi(0)}{\mu_0 q(0) R_0} = 1.06 \times 10^6 \text{ A m}^{-2}$$

$$\boxed{P_\Omega = 996.8 \text{ W m}^{-3}}$$

2. a.

$$\lambda_{crit} = \frac{12400}{0.63 \times 300} = 65.6085\mathring{A}$$

Then $\omega_{pe} = 2\pi c \lambda_{crit}^{-1} = 2.87 \times 10^{17}$ s^{-1} and, since $\omega_{pe}^2 = n_e e^2/(m_e \epsilon_0)$, we have $\boxed{n_e = 2.59959 \times 10^{31}\text{m}^{-3}}$

2. b.

$\boxed{\rho = nm_i = 108.316 \text{ g cm}^{-3}}$, $\boxed{\rho/\rho_{liq} = 433.264}$

2. c.

$M = (4/3)\pi\rho R^3 = (4/3)\pi(\rho R)^3/\rho^2 = 357\mu$g. Then
$f_B = \rho R/(6 + \rho R) = 0.142857$ and $Y'' = 3.39 \times 10^{11}$J g^{-1}. $Y = MY''$ gives

$$\boxed{\text{Y} = 17.29 \text{ MJ}}$$

2. d.

$q''_{BB} = \sigma T^4 = 1.03 \times 10^5 (300)^4 = 8.343 \times 10^{14}$ W m^{-2}. Setting
$\sigma T^4 \cdot A = (1/2)U_{laser}/\tau$ gives

$$\boxed{U_{laser} = 2.35 \text{ MJ}}$$

2. e.

$$\boxed{Q = Y/U_{laser} = 7.33}$$

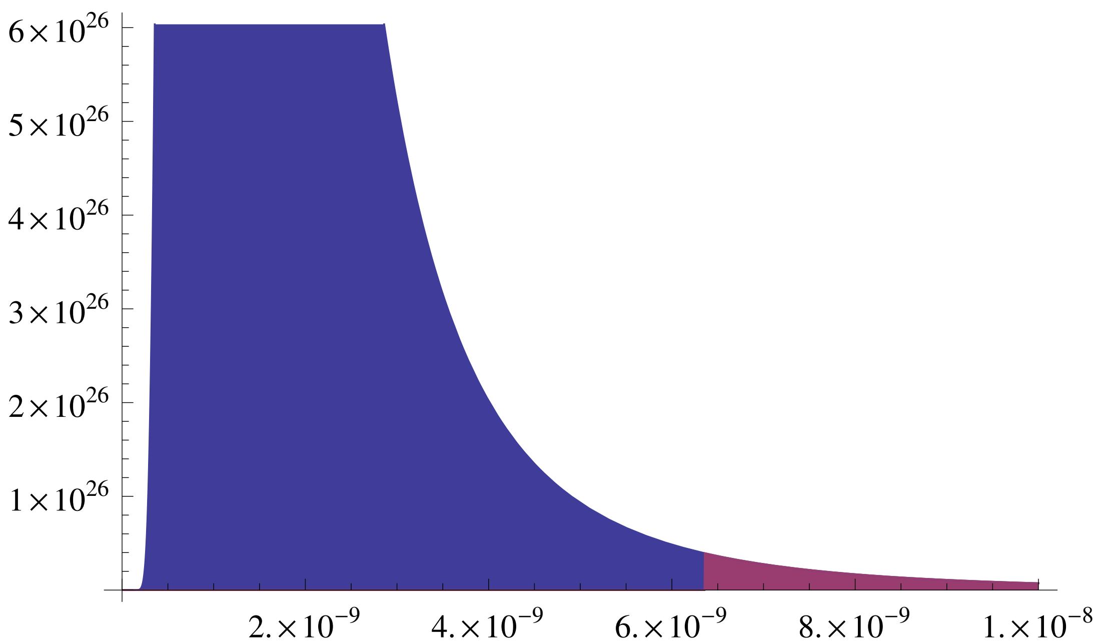

Figure 1: Blackbody Radiation Curve, showing cutoff portion.