ME106 – Midterm#1 – Problem 1 solution

a/ u_θ : circumferential velocity at radial position r.

Ωr_2: tangential velocity at r=r$_2$ due to rotation of outer cylinder.

So the LHS is the ratio of circumferential velocity at r to the tangential velocity at r$_2$.

$[u_\theta] = L.T^{-1}$, $[\Omega r_2] = T^{-1}.L$

$[\frac{1}{1-\kappa^2}(\frac{r}{r_2} - \kappa^2 \frac{r_2}{r})] = I$ as $[\kappa] = I$ ("I" stands for 'no dimension', or $L^o M^o T^o$).

b/ At r=r$_1$, $\frac{u_\theta}{\Omega r_2} = \frac{1}{1-\kappa^2}(\frac{r_1}{r_2} - \kappa^2 \frac{r_2}{r_1})$ =0 (Thus, the Inner cylinder is fixed).

At r=r$_2$, $\frac{u_\theta}{\Omega r_2} = \frac{1}{1-\kappa^2}(\frac{r_2}{r_2} - \kappa^2 \frac{r_2}{r_2})$ =1.

The fluid particles located at r=r$_1$ or r=r$_2$ have the same velocity as the tangential velocity at the walls of the cylinders. The 'no-slip' condition is satisfied. The fluid "sticks" to the surface of the cylinders because it has viscosity.

c/ Shear stress at r=r$_1$ (assuming that $\frac{\partial}{\partial y} = \frac{\partial}{\partial r}$):

$$\tau_1 = \mu \frac{\partial u_\theta}{\partial r}\Big|_{r=r1} = \mu \frac{\Omega r_2}{1-\kappa^2}[\frac{\partial}{\partial r}(\frac{r}{r_2}) - \kappa^2 \frac{\partial}{\partial r}(\frac{r_2}{r})]\Big|_{r=r1} = \mu \frac{\Omega r_2}{1-\kappa^2}[\frac{1}{r_2} + \kappa^2 \frac{r_2}{r^2}]\Big|_{r=r1}$$

Since: $\frac{\partial}{\partial r}(\frac{1}{r}) = \frac{-1}{r^2}$, $\tau_1 = \mu \frac{\partial u_\theta}{\partial r}\Big|_{r=r1} = \mu \frac{\Omega r_2}{1-\kappa^2}[\frac{1}{r_2} + \kappa^2 \frac{r_2}{r_1^2}] = \frac{2\mu\ \Omega}{1-\kappa^2}$

d/ Shear stress at r=r$_2$

$$\tau_2 = \mu \frac{\partial u_\theta}{\partial r}\Big|_{r=r2} = \mu \frac{\Omega r_2}{1-\kappa^2}[\frac{1}{r_2} + \kappa^2 \frac{r_2}{r_2^2}] = \frac{2\mu\ \Omega}{1-\kappa^2}(\frac{1+\kappa^2}{2})$$

But $r_1 < r_2 \Rightarrow \kappa < 1 \Rightarrow \tau_1 > \tau_2$

e/ Torque on the inner cylinder: $T = r_1.\tau_1.A_1 = \frac{4\pi\mu\ \Omega r_1^2}{1-\kappa^2}$

Torque (or torsion) is defined as the product of a force by a lever arm (here this arm is r$_1$). Shear stress is a force per unit area. The area A$_1$ is the SURFACE AREA associated with the perimeter of the inner cylinder. Since the problem is per unit length (in z–direction) of the cylinder axis, A$_1$ is reduced to the circumference of the inner cylinder.

Numerical computations: From chart, $\mu = 0.4$ N.s/m^2 (Log scale on the vertical axis !!); Also, κ=0.667, r$_1$ = 0.2m, Ω=10 rad/s, (Observe units are put into equation to check the units of the desired quantity).

$$T = r_1.\tau_1.A_1 = \frac{4\pi \times 0.4\ \ N.s/m^2 \times 10 rad/s \times (.2m)^2}{1-.667^2}$$ =3.6 N-m / m (of z)

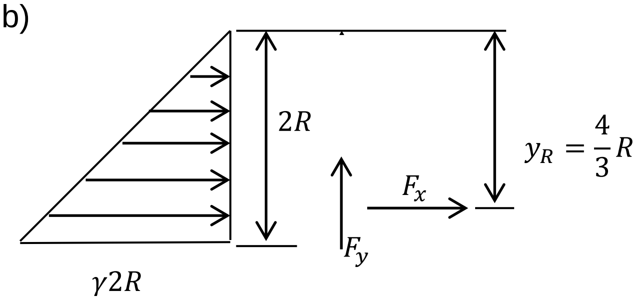
Problem 2 Solution

a)

$$\left[\frac{F_x}{\gamma R^2}\right] = \frac{\frac{ML1}{T^2L}}{\frac{M}{L^3}\frac{L}{T^2}L^2} = \frac{\frac{M}{T^2}}{\frac{M}{T^2}} = I \qquad \left[\frac{F_y}{\gamma R^2}\right] = \frac{\frac{ML1}{T^2L}}{\frac{M}{L^3}\frac{L}{T^2}L^2} = \frac{\frac{M}{T^2}}{\frac{M}{T^2}} = I \qquad \left[\frac{M_o}{\gamma R^2}\right] = \frac{\frac{ML\,L}{T^2L}}{\frac{M}{L^3}\frac{L}{T^2}L^3} = \frac{\frac{M}{T^2}}{\frac{M}{T^2}} = I$$

$$I \text{ is dimensionless}$$

b)



$$F_x = \gamma h_c A = \gamma y_c 2R = \gamma 2R^2$$

$$(A = 2R * \text{unit length in } z, \text{normal to paper})$$

$$\frac{F_x}{\gamma R^2} = 2$$

$$M_o{}^{(x)} = F_x(2R - y_R) = \gamma 2R^2\left(2R - \frac{4}{3}R\right) = \gamma\frac{4}{3}R^3$$

$$\frac{M_o{}^{(x)}}{\gamma R^3} = \frac{4}{3}$$

c) Vertical Force due to weight of water on top of structure

Area of Wedge (A_w) = Area of Square R x R (A_{sq}) – ¼ Area of Circle with radius R (A_{qc})

$$A_w = A_{sq} - A_{qc} = R^2 - \frac{1}{4}\pi R^2 = R^2\left(1 - \frac{1}{4}\pi\right)$$

$$F_y = -W = -A_w\gamma = -\gamma R^2\left(1 - \frac{1}{4}\pi\right)$$

$$\frac{F_y}{\gamma R^2} = -\left(1 - \frac{1}{4}\pi\right)$$

Need to solve for the Centroid of the Wedge to find the point of action for the F_y

$$A_w x_{c\_w} = A_{sq}x_{c\_sq} - A_{qc}x_{c\_qc}$$

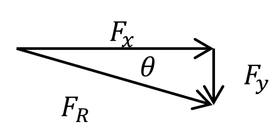
[Note: centroid values were on reference sheets on the Midterm Quiz]

$$x_{c\_w} = \frac{A_{sq}x_{c\_sq} - A_{qc}x_{c\_qc}}{A_w}$$

$$x_{c\_w} = \frac{R^2\frac{1}{2}R - \frac{1}{4}\pi R^2\left(R - \frac{4R}{3\pi}\right)}{R^2\left(1 - \frac{1}{4}\pi\right)} \quad ==> \quad x_{c\_w} = \frac{R\left(\frac{5}{6} - \frac{\pi}{4}\right)}{\left(1 - \frac{1}{4}\pi\right)}$$

$$M_o{}^{(y)} = F_y x_{c\_w} = \frac{R\left(\frac{5}{6} - \frac{\pi}{4}\right)}{\left(1 - \frac{1}{4}\pi\right)}\gamma R^2\left(1 - \frac{1}{4}\pi\right) \quad ==> \quad \frac{M_o{}^{(y)}}{\gamma R^3} = \left(\frac{5}{6} - \frac{\pi}{4}\right)$$

e) $\tan(\theta) = \dfrac{\frac{F_y}{\gamma R^2}}{\frac{F_x}{\gamma R^2}} = \dfrac{-\left(1 - \frac{1}{4}\pi\right)}{2} \cong -0.1073 => \theta = -6.12^o$



Problem 3 Solution:

$\forall_1, \forall_2$ are the volumes of the rubber chamber, not including the tube

v_1, v_2 are the specific volumes of configuration 1,configuration 2.

Dimensions $[v] = \frac{L^3}{M}$

$P_1 v_1 = P_2 v_2 (= RT, isothermal\ process)$

$$=> \frac{v_1}{v_2} = \frac{\forall_1}{\forall_2}\ if\ no\ mass\ enters\ system \quad ==> \quad P_2 = P_1 \frac{\forall_1}{\forall_2}$$

$$P_2 + 13.3\gamma_{h20} h = P_1 \frac{\forall_1}{\forall_2} + 13.3\gamma_{h20} h = P_1 \quad \Rightarrow \quad \frac{\forall_1}{\forall_2} = 1 - \frac{13.3\gamma_{h20} h}{P_1}$$

$$\frac{\forall_2}{\forall_1} = \frac{P_1}{P_1 - 13.3\gamma_{h20} h}$$

b) γ_{H20}= 62.4 lb/ft^3, P_{atm}=14.7 lb/in^2

$P_1 = P_{abs}$= 14.7lb/in^{2}h = 6in = ½ ft

$$\frac{\forall_1}{\forall_2} = 1 - \frac{13.3\gamma_{h20} h}{P_1} \qquad \frac{\forall_1}{\forall_2} = 0.8039 \quad \frac{\forall_2}{\forall_1} = 1.2439$$

c) $R^o = 60^o + 459.67^o = 519.67^o$ $\quad \forall_1$= 0.012 ft^3 $\qquad P_1 = P_{abs}$=14.7lb/in^2

$$P_1 v = RT \qquad v = \frac{RT}{P_1} \qquad \frac{\forall_1}{m} = \frac{RT}{P_1} \qquad m = \frac{P_1 \forall_1}{RT}$$

$$m = \frac{14.7\frac{lb}{in^2} * 144\frac{in^2}{ft^2} * 0.012 ft^3}{1.71x10^3 \frac{lb\ ft}{slug R^o} * 519.67 R^o} = 28.58x10^{-6} slugs$$