MATH 54, midterm test, Fall 2010.

1. Let $W := \{f \in \mathbb{P}_3 : f(2) = f''(2)\}$. Is W a subspace of $\mathbb{P}_3$?

2. Let

$$A := \begin{bmatrix} 2 & 3 & 4 \\ -2 & -3 & -4 \\ 2 & 1 & 2 \\ -2 & -1 & -2 \end{bmatrix} \qquad \text{and} \qquad \vec{v} := \begin{bmatrix} 2 \\ -2 \\ 2 \\ 2 \end{bmatrix}.$$

(a) Is the linear system $A\vec{x} = \vec{v}$ solvable?
(b) Find the distance between $\vec{v}$ and the column space $\mathrm{Col}(A)$ of A.

3. Let P be the parallelogram in $\mathbb{R}^2$ determined by $(-3, 1)$ and $(1, 1)$. Let T be the linear map with standard matrix

$$\begin{bmatrix} 3 & 2 \\ 0 & -1 \end{bmatrix}.$$

Find the area of the figure $T(P)$ and sketch its graph.

4. Mark each of the following true or false. Provide short explanations.

(a) The nonzero rows of a matrix form a basis for its row space.

(b) Elementary row operations on a matrix can change its null space.

(c) The nonpivot columns of a matrix are always linearly dependent.

(d) If A is an $m \times n$ matrix and the linear transformation $\vec{x} \mapsto A\vec{x}$ is onto, then $\operatorname{rank} A = m$.

(e) If $\mathcal{B} = \{\vec{b}_1, \ldots, \vec{b}_n\}$ and $\mathcal{C} = \{\vec{c}_1, \ldots, \vec{c}_n\}$ are bases for a vector space V, then the jth column of the change-of-coordinates matrix $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P}$ is the coordinate vector $[\vec{c}_j]_{\mathcal{B}}$.

5. Let

$$A := \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}, \quad \vec{x}_0 := \begin{bmatrix} 4 \\ -1 \end{bmatrix}, \quad \text{and let} \quad \vec{x}_n := A\vec{x}_{n-1}, \quad n = 1, 2, \ldots .$$

Find $\lim_{n\to\infty} \vec{x}_n$.

6. Classify the quadratic form $-5x_1^2 + 4x_1x_2 - 2x_2^2$: is it positive definite, positive semidefinite, negative definite, negative semidefinite, or indefinite?