Spring 2010 MSE 111

Midterm Exam

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3/16/10, 9:40 am

80 minutes, 74 points total, 10 pages

Name:______________________________________

SID:______________________________

Problem	1	2	3	4	5	6	7	TOTAL
Points Possible	5	16	12	10	7	8	16	74
Points								

SHOW ALL OF YOUR WORK!!!

Answers given without supporting calculations will be marked wrong, even if they are numerically correct.

1. (5 pts.) True/False

a) *True or* ***False*** The number of free electrons in a metal increases according to the Boltzmann distribution.

b) *True or* ***False*** The position and momentum of an electron can be known precisely at the same time.

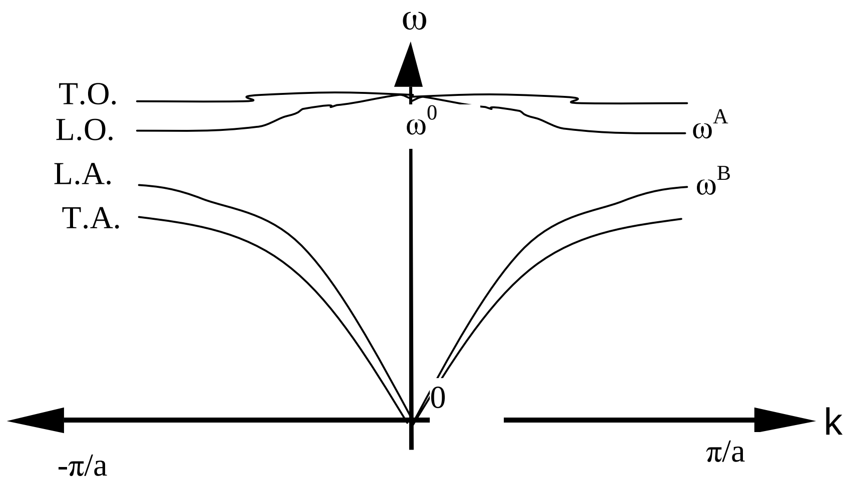
c) *True or* ***False*** SiC has purely covalent bonds.

d) *True or* ***False*** Bonding electrons in Cr have localized character.

e) ***True*** *or False* Deep level dopants are more delocalized in k-space compared to shallow level dopants.

2. (16 points) Phonons

a) (4 pts.) Neatly sketch the dispersion relation for phonons in a 1-D chain of two different types of atoms (i.e., …K-Br-K-Br…) on the axes below.



b) (4 pts.) Given the following three expressions, label all endpoints on your sketch in *a* for all *longitudinal* phonon frequencies (i.e., both optical and acoustic if both are present).

$$\omega^A = \sqrt{\frac{2C}{M_{lighter\_atom}}} \qquad \omega^B = \sqrt{\frac{2C}{M_{heavier\_atom}}} \qquad \omega^o = \sqrt{2C\left(\frac{1}{M_1}+\frac{1}{M_2}\right)}$$

c) (3 pts.) Assuming a lattice constant of a, label the minimum and maximum values of k on your sketch. Why did you choose these values (one short sentence)?

±π/a because this is the edge of the Brillouin Zone. Outside of these values indicates waves shorter than the distance between reciprocal lattice points which mean nothing

d) (5 points) Now, given that the optical frequency at $k/k_{max} = 1$ is $4\text{x}10^{12}$ Hz for a KBr chain, calculate the optical frequency at k=0 for KBr. The atomic masses are 39 for K and 80 for Br.

$$\omega_A = 4x10^{12}\,Hz = \sqrt{\frac{2C}{39}} \Rightarrow C = 3.12x10^{26}$$

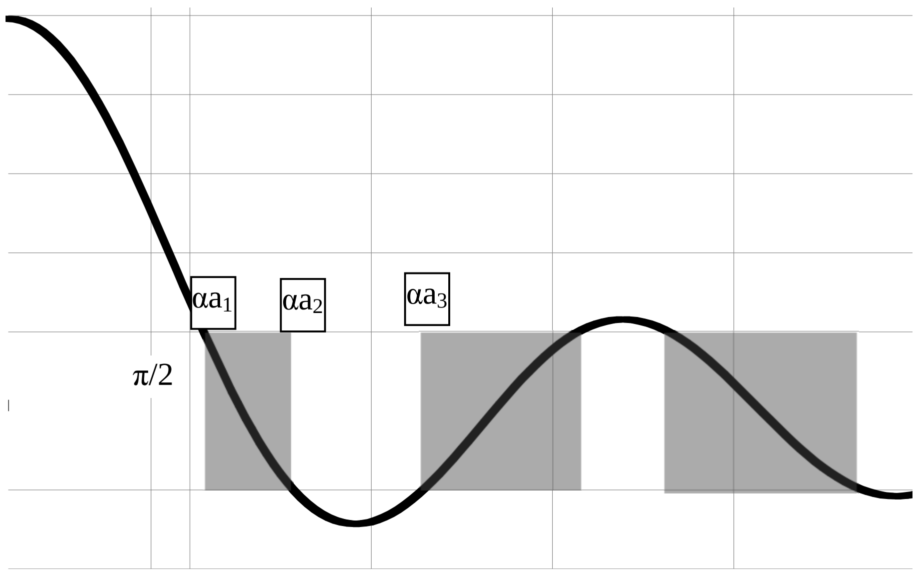
$$\omega_0 = \sqrt{2C\left(\frac{1}{80}+\frac{1}{39}\right)} = 4.88x10^{12}\,Hz$$

(Don't ding them twice if they labeled the above graph wrong. You can assume math is ok.)

3. (12 points) Kronig Penney

A quantum well structure is grown with 2Å GaAs layers sandwiched between 5 Å $Al_{1-x}Ga_xAs$ layers. This means that the well width, a, is 2Å while the well spacing, w, is 5Å. The following expressions and plot are provided to answer parts a-d. You may assume that the effective mass of an electron in this structure is 9.1×10^{-31} kg. Make your procedure obvious since you will be reading numbers off of a graph.

$$F(\alpha a) = P\frac{\sin(\alpha a)}{\alpha a} + \cos(\alpha a) \qquad P = \frac{m^* a}{\hbar^2} V_0 w \qquad \alpha = \sqrt{\frac{2m^* E}{\hbar^2}}$$



a) (4 pts.) Calculate the height of the well, V_0, in eV using the plot. Show your work.

$$F(\pi/2) = (fromgraph) 2.6 = P\frac{1}{\pi/2} + 0 \Rightarrow P = 4.084$$

$$V_0 = \frac{P\hbar^2}{mawe} = 3.12eV$$

b) **(4 pts.)** Shade the allowed minibands on the plot above.

See plot

c) **(4 pts.)** What is the width of the first allowed energy band and first disallowed energy gap in eV?

$$\frac{\alpha^2\hbar^2}{2me} = E(\alpha) where \alpha = \alpha a / a$$

αa_1=2.25 -> E1=4.82eV
$\alpha a2$=3.1->E2=9.17eV
αa_3=4.5->E3=19.31

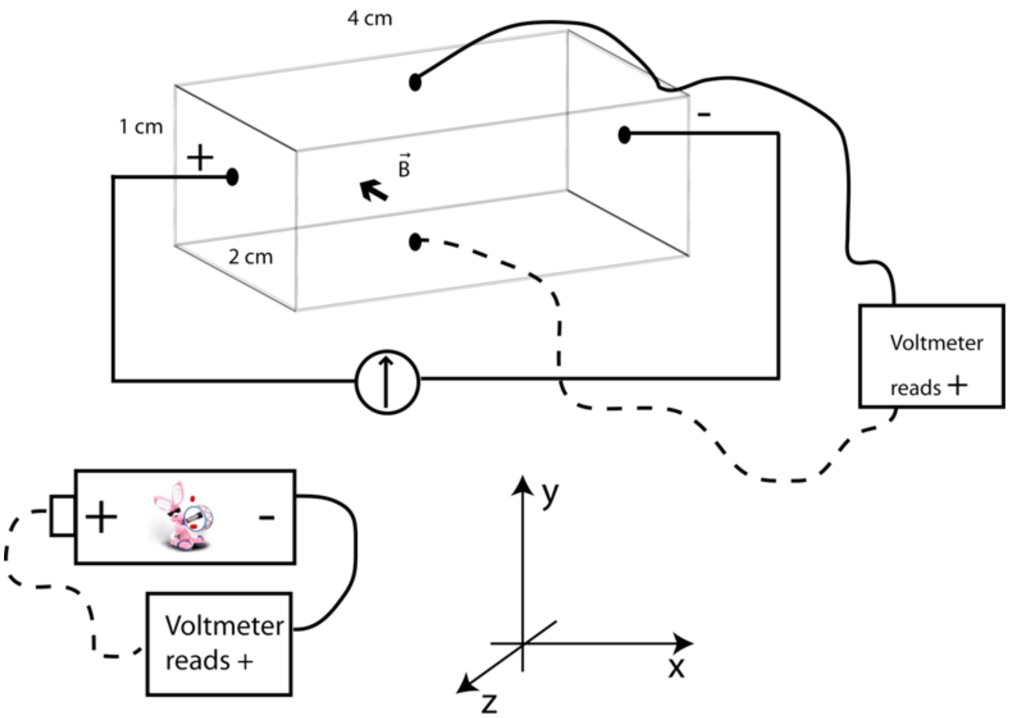
$W_{allowed}$=9.17-4.82=4.35eV
$W_{disallowed}$=10.14eV

Extra credit (+2 pts.): Suggest a quantitative cutoff for the minimum energy gap to be observable at 300K.

0.0259eV = kT

4. Hall Effect (10 points)

The setup shown below is used to measure the electrical characteristics of a piece of Si. The 1 Tesla magnetic field points INTO the page (-z direction) and the 1 mA applied current flows in the +x direction.



a) (2 pts.) In which direction would holes be deflected due to the Hall effect? In which direction would electrons be deflected?

	Direction deflected
Holes	Up or +y
Electrons	Up or+y

b) (2 pts.) Now, considering the voltmeter reading of + 15 mV, is the Si sample n-type or p-type?

n-type

c) (6 pts.) Assuming a carrier effective mass of 0.2 and resistivity of 0.02 Ω-cm, what do you expect the mobility and lifetime of a carrier in this sample to be? (Hint: check out the equation sheet and clearly show your work.)

$$n = \frac{JB}{E_H e} where J = \frac{0.001A}{0.01m x 0.2m} = 5A/m^2 and E_H = \frac{0.015V}{0.02m} = 0.75V/m$$

$$n = 4.17x10^{19} m^{-3}$$

$$\mu = \frac{1}{\rho en} = \frac{1}{0.0002\Omega m \cdot 4.17x10^{19} m^{-3} \cdot e} = 749m^2/Vs$$

$$\tau = \frac{\mu m}{e} = \frac{749 \cdot 0.2 \cdot 9.1x10^{-31}}{1.6x10^{-19}} = 8.5x10^{-10} \sec or 0.85ns$$

5. (7 pts.) Density of States

Derive the density of states, Z(E), for electrons in a very thin metal film. You should treat the system as two dimensional. This is not just an academic exercise, as there are materials that have two dimensional properties. Quantum wells and graphene (a single sheet of graphite) are two examples, although the physics is a bit more complicated.

In the free electron model, the 2D system can be represented as a thin film with sides of length ***a****. These* ***a*** *define the length of a 2D infinite potential well. For a 2D infinite potential well, the energy is given by:*

$$E = \frac{h^2}{8m_e a^2}\left(n_1^2 + n_2^2\right)$$

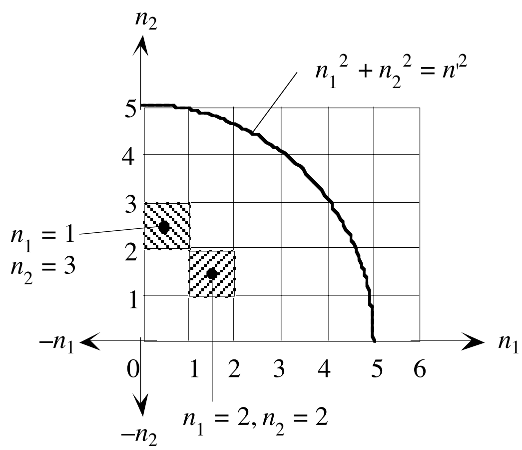
Only positive n_1 and n_2 are allowed. Each n_1 and n_2 combination is an orbital state. We can define a new variable n as:

$$n^2 = n_1^{\ 2} + n_2^{\ 2} \qquad \text{Therefore}\rightarrow \quad E = \frac{h^2}{8m_e a^2} n^2$$

Let us consider how many states there are with energies less than E'. E' corresponds to $n \le n'$.

$$E' = \frac{h^2}{8m_e a^2} n'^2 \qquad \therefore\ n' = \sqrt{\frac{8a^2 m_e E'}{h^2}}$$

Each state, or electron wavefunction in the crystal, can be represented by a box at n_1, n_2.



From the previous figure, we know that all states within the quarter arc defined by n' have $E < E'$. The area of this quarter arc is the total number of orbital states. The total number of states, S, including spin is twice as many,

$$S = 2\left(\frac{1}{4}\pi n'^2\right) = 2\left(\frac{1}{4}\pi\left[\sqrt{\frac{8a^2 m_e E'}{h^2}}\right]^2\right)$$

$$\therefore \qquad S = \frac{4\pi a^2 m_e E'}{h^2}$$

The density of states g is defined as the number of states per unit area per unit energy. Therefore,

$$g = \frac{1}{a^2}\frac{dS}{dE'} = \frac{1}{a^2}\frac{4\pi a^2 m_e}{h^2} = \frac{4\pi m_e}{h^2}$$

From the previous result we can conclude that, for a two dimensional system, the density of states is constant.

6. (8 pts.) Semiconductor Statistics

a) (2 pts.) Write down the form of the Fermi distribution function for holes

$$f_{holes}(E) = 1 - \frac{1}{1 + Exp\left[\frac{E - E_F}{k_b T}\right]}$$

b) (2 pts.) Simplify the above equation in the case of E – E_F >> kT.

$$f_{holes}(E) = 1 - \frac{1}{1 + Exp\left[\frac{E - E_F}{k_b T}\right]} \rightarrow f_{holes}(E) = 1 - Exp\left[-\frac{E - E_F}{k_b T}\right]$$

c) (4 pts.) At what temperature is there a 5 percent probability that a state with an energy 0.1 eV below the Fermi energy will be occupied by a hole?

$$f(E) = \frac{1}{1 + Exp\left[\frac{E - E_F}{k_b T}\right]} \rightarrow 0.95 = \frac{1}{1 + Exp\left[-\frac{0.1}{8.62 \times 10^{-5} T}\right]} \rightarrow T = 394K$$

Alternatively,

$$f_{holes}(E) = 1 - \frac{1}{1 + Exp\left[\frac{E - E_F}{k_b T}\right]} \rightarrow 0.05 = 1 - \frac{1}{1 + Exp\left[-\frac{0.1}{8.62 \times 10^{-5} T}\right]} \rightarrow T = 394K$$

7. (16 pts.) Semiconductors

a) (4 pts.) In intrinsic semiconductors the concentration of free carriers is a strong function of temperature. The intrinsic carrier concentration in Ge (E_g = 0.66 eV) at 300 K (room temperature) is 3×10^{13} cm^{-3}. Ignore the temperature dependencies of E_g, N_C, and N_V. At what temperature will the concentration be two orders of magnitude lower?

$$n_i = \sqrt{N_c N_v}\exp\left(-\frac{E_g}{2k_b T}\right) \to 3\times10^{13} = \sqrt{N_c N_v}\exp\left(-\frac{0.66}{2\times8.62\times10^{-5}\times300}\right)$$

$$\sqrt{N_c N_v} = 1.045\times10^{19}\,cm^{-3}$$

$$\frac{3\times10^{13}}{100} = 1.045\times10^{19}\exp\left(-\frac{0.66}{2\times8.62\times10^{-5}T}\right) \to T = 220K$$

b) (6 pts.) Semiconductor Doping

Fill in the tables below by specifying the impurity type in Si and in AlAs. For AlAs, give an answer for both atomic sites. Use acceptor = **A**, double acceptor = **2A**, donor = **D,** double donor = **2D**, or neutral = **N.** Mark an **X** for dopants that would be triple donors or acceptors**.**

Dopant	**Al site**	**As site**
Ge in AlAs	*D*	*A*
Ga in AlAs	*N*	*2A*
Te in AlAs	*X*	*D*
Mg in AlAs	*A*	*X*

Dopant	**Type**
Sb in Si	*D*
Al in Si	*A*
Be in Si	*2A*
P in Si	*D*

c) (2 pts.) *Briefly* explain the difference between extrinsic and intrinsic semiconductors.

An intrinsic semiconductor is a material with no impurity atoms present in the crystal. An extrinsic semiconductor is defined as a semiconductor in which dopants or impurity atoms have been added.

d) (2 pts.) In one sentence, what is a phonon?

A phonon is a quantized lattice vibration

e) (2 pts.) Within the framework of the hydrogenic model, can you get the carrier effective mass from a plot of E vs. k of a given semiconductor? If so, how? If not, why not?

Yes, you can. The effective mass is proportional to the inverse of the second derivative of the E/k parabola.

Constants and Equations:

$m_e = 9.11x10^{-31} kg$ $\hbar = 1.054x10^{-34} J \cdot s$ $h = 6.625x10^{-34} J \cdot s$

$k_b = 8.62x10^{-5} eV / K$ $e = -1.6x10^{-19} C$ $c = 3x10^{8} m/s$

$1eV = 1.6x10^{-19} J$ $N_A = 6.02x10^{23} atoms/mole$ $\sin\left(\frac{\pi}{2}\right) = 1$

$\Delta x \cdot \Delta p \geq \hbar / 2$ $F = m \cdot a$

$\lambda = \frac{h}{mv} = \frac{h}{p} = \frac{2\pi}{k}$ $n\lambda = 2d\sin\theta$

$E = \frac{p^2}{2m} = \frac{1}{2}mv^2$ $E = \frac{hc}{\lambda} = h\nu$ $E_n = \frac{n^2h^2}{8mL^2}$

$R = \rho\frac{L}{A}$ $\mu = \frac{e\tau}{m}$

$\sigma = e(n_e\mu_e + n_h\mu_h)$ $\sigma = \frac{1}{\rho}$

$J = N_e v_d e = N\ \mu\ Ee$ $E_H = R_H JB$ $R_H = \frac{1}{Ne}$

time-independent Schrödinger equation:

$$\left(\frac{-\hbar^2}{2m}\right)\frac{\partial^2\psi}{\partial x^2} + V\psi = E\psi$$

$n_i = \sqrt{N_c N_v}\exp\left(-\frac{E_g}{2k_bT}\right)$ $N_v = 2\left(\frac{2\pi\, m_h^* k_b T}{h^2}\right)^{3/2}$ $N_c = 2\left(\frac{2\pi\, m_e^* k_b T}{h^2}\right)^{3/2}$

$n = N_c\exp\left(-\frac{E_g - E_F}{k_bT}\right)$ $p = N_v\exp\left(-\frac{E_F}{k_bT}\right)$

$$\omega = \left(\frac{4C}{M}\right)^{1/2}\left|\sin\frac{ka}{2}\right|$$