Engineering 117
Fall Semester 2010
First Midterm Examination

October 5, 2010

SEVENTY-FIVE MINUTES, CLOSED BOOK. ONE: $8\frac{1}{2}'' \times 11''$ SHEET OF NOTES ALLOWED.

1. A differential equation for a function $y(x)$ is given by

$$y''(x) - 2y'(x) + y(x) = 8\ \cosh(x)$$

   (a) How many initial conditions are required to find the complete solution for $y(x)$?

   (b) How many independent solutions to the homogeneous equation $y''(x)-2y'(x)+y(x) = 0$ must be found? What are these solutions?

   (c) Does the right hand side of the equation contain terms which are a solution to the homogeneous equation ? If so, what would you take as a form $y_p(x)$ for the particular response? (Hint: write out cosh in terms of elementary exponential functions.)

   (d) Solve for for $y_p(x)$.

   (e) For initial conditions $y(0) = 0$ and $y'(0) = -1$, solve for the complete response.

2. A system of ODEs is written in the form

$$\frac{d\mathbf{y}(t)}{dt} = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \mathbf{y}(t)$$

   (a) Solve for the eigenvalues (λ_1, λ_2) of the matrix operating on $\mathbf{y}(t)$ on the RHS and then write down a general solution for $\mathbf{y}(t)$ in terms of two independent vectors $\mathbf{y}_1$ and $\mathbf{y}_2$, yet to be determined.

   (b) Now solve for the two vectors $\mathbf{y}_1$ and $\mathbf{y}_2$ by using the eigenvalues calculated in part (a).

   (c) If the initial condition is $\mathbf{y}(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, find $\mathbf{y}(t)$ for all time $t > 0$.

3. A differential equation for a forced system is given by

$$y''(t) + 3y'(t) + 2y(t) = f(t)$$

where

$$f(t) = \begin{cases} \cos t - 3\sin(t) & t \geq 0 \\ 0 & t < 0 \end{cases}$$

(a) Write down a Laplace transform equation in the form $\mathbf{G}(s)\mathbf{Y}(s) + \mathbf{IC}(s) = \mathbf{F}(s)$, where $\mathbf{GY} + \mathbf{IC}$ and $\mathbf{F}$ are the transforms of the LHS and RHS, respectively. Leave the initial conditions $\mathbf{IC}$ as arbitrary at this point.

(b) For initial conditions $y(0) = 1$ and $y'(0) = 0$, solve for $\mathbf{Y}(s)$. Write this as an expanded partial fraction.

(c) Find the complete solution $y(t)$ for all times $t > 0$.