Engineering 115: Engineering Thermodynamics

First Midterm Exam

INSTRUCTIONS: You are allowed one side of a standard piece of paper with relevant equations. Answer all questions and show the steps in your work to maximize credit.

1. Considering the first and second laws of thermodynamics, and the formal mathematical relations between state functions, answer the following questions. For each problem we are considering closed systems. Do not assume the systems are ideal gasses.

   a. (10 points) A system undergoes an isothermal transformation at T=300 K, involving an internal energy change of $\Delta U=400$ J and an entropy change of $\Delta S=2$ J/K. What is the maximum work possible in this transformation?

   b. (10 points) A system undergoes a change of state at a constant pressure of P=1 atm. In the process we add q=30 J of heat to the system, while we perform w=40 J of work on the system. What is ΔU, ΔH and ΔV for this change of state?

   c. (10 points) Consider a system that has a negative coefficient of thermal expansion:

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P < 0$$

   The system undergoes a constant temperature process in which the pressure increases. Does the entropy of the system increase, decrease or stay the same?

Midterm 1: Solutions

①

(a) $\Delta U = q - W$ 1st Law

$\frac{\delta q}{T} \leq dS$ 2nd Law

$\Downarrow$

$\Delta S \geq \frac{q}{T} \Rightarrow T\Delta S \geq q$ for isothermal reaction

Δ [illegible]

$W = q - \Delta U \leq T\Delta S - \Delta U$

max work for reversible transformation

$W_{max} = T\Delta S - \Delta U = 300K \cdot 2\frac{J}{K} - 400J$

$$\boxed{W_{max} = 200J}$$

(b) $\Delta U = q - W$

$\Delta H = q$

~~$W = P\Delta V \Rightarrow$~~

$\Delta V = W/P$

Note: we perform work, so $W < 0$

$\Rightarrow W = -40J$

$$\Delta U = 30 + 40 = 70\ J$$

$$\Delta H = 30\ J$$

$$\Delta V = \frac{-40\,\cancel{J}}{1\,atm} \times \frac{1\ \ell\ \cancel{atm}}{101.3\,\cancel{J}} = -0{,}395\ \ell$$

(c) $$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$$ by Maxwell relation

$$\left(\frac{\partial S}{\partial P}\right)_T = -\alpha V > 0$$

$\Rightarrow$ entropy <u>increases</u>

2. The following is the so-called Diesel cycle for a compression-ignition heat engine. The cycle involves four reversible processes:

   (i) 1→2 Adiabatic compression
   (ii) 2→3 Constant-pressure heat addition
   (iii) 3→4 Adiabatic expansion
   (iv) 4→1 Constant-volume heat rejection

   Suppose we have 5 moles of a diatomic ideal gas operating in a Diesel cycle. In the initial state the gas is at standard temperature and pressure. The maximum temperature reached during the cycle is 700 K, and the minimum volume is 60 liters. The molar constant-volume heat capacity for a diatomic gas is $c_V=5/2\ R$.

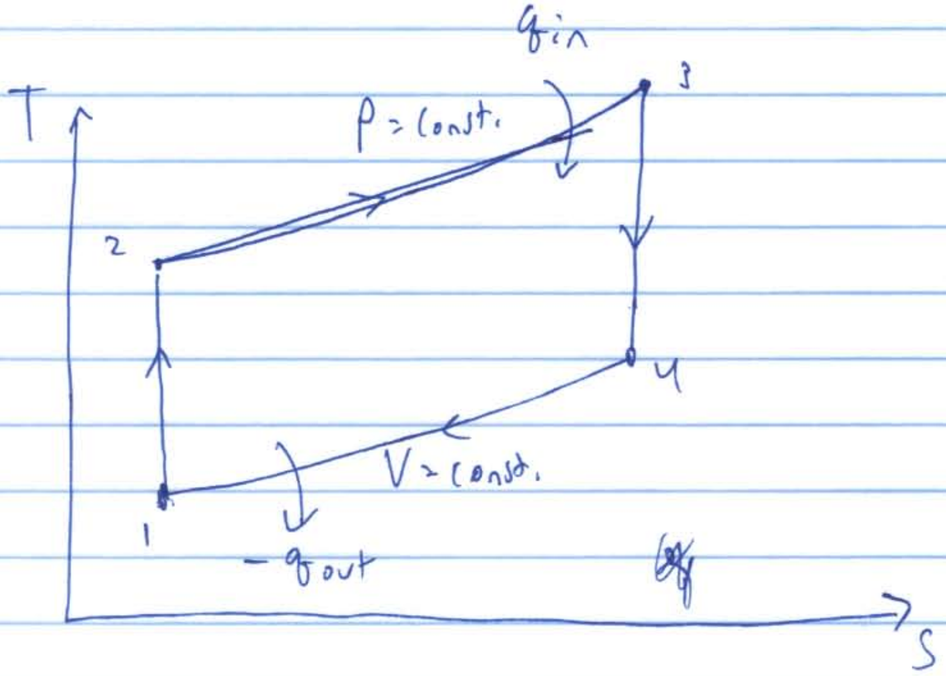
   a. (20 points) Plot the cycle on a T-S and P-V diagram. On your diagram label states 1, 2, 3 and 4. Indicate on your plot the step during which heat q_{in} is added to the system, and the step where heat in the amount $-q_{out}$ is rejected by the system.

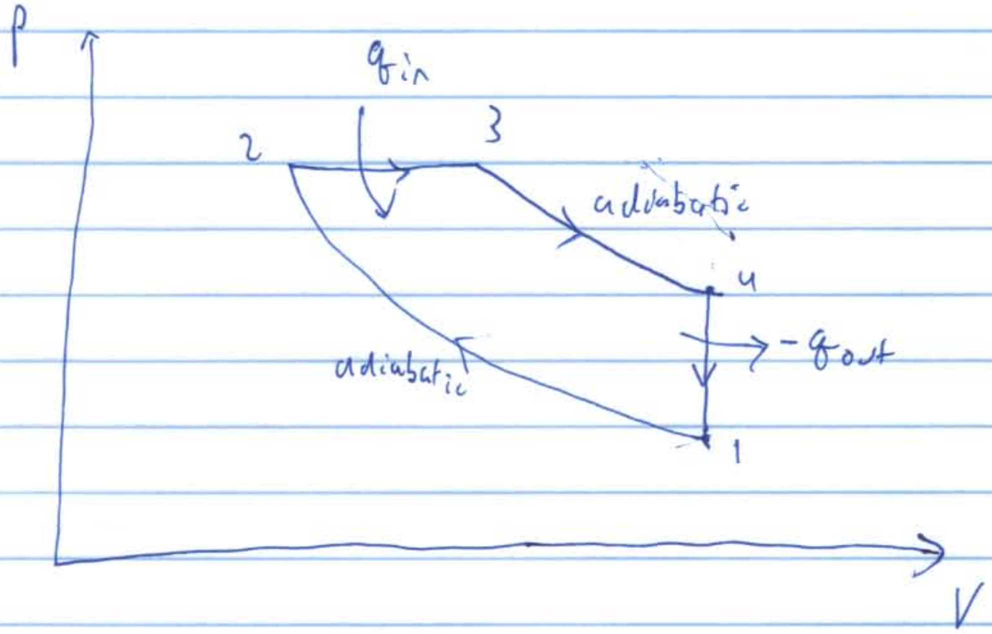
   b. (20 points) Compute the total work done in the cycle and the thermal efficiency.

   c. (10 points) Suppose that the maximum temperature in the cycle is increased, keeping the minimum volume and the initial state the same. Would you expect the work to increase or decrease? Explain your answer.

2.

(a)


T
q_{in}
3
$P = const.$
2
4
$V = const.$
1
$-q_{out}$
S


P
q_{in}
2
3
adiabatic
4
adiabatic
$-q_{out}$
1
V

2. $c_v = 5/2\ R$ $c_p = 7/2\ R$ $\gamma = c_p/c_v = 7/5$

(b) $P_1 = 1\,atm$
$T_1 = 300\,k$
$V_1 = nRT/P_1 = 123\ l$
$n = 5$ moles

$P_2 = nRT_2/P_2 = 2.74$ at
$V_2 = 60\ l$
$T_2 = (V_1/V_2)^{\gamma-1} T_1 = 400\ k$

$P_3 = P_2 = 2.74\ atm$
$V_3 = 105\ l$
$T_3 = 700\,K$

$P_4 = 2.20\ atm$
$V_4 = 123\ l$
$T_4 = (V_3/V_4)^{\gamma-1} T_3 = 657$

$W_{tot} = q_{in} - q_{out}$ by 1st law

$$W_{tot} = n c_p (T_3 - T_2) + n c_v (T_1 - T_4)$$

$$= \frac{35}{2} R (T_3 - T_2) + \frac{25}{2} R (T_1 - T_4)$$

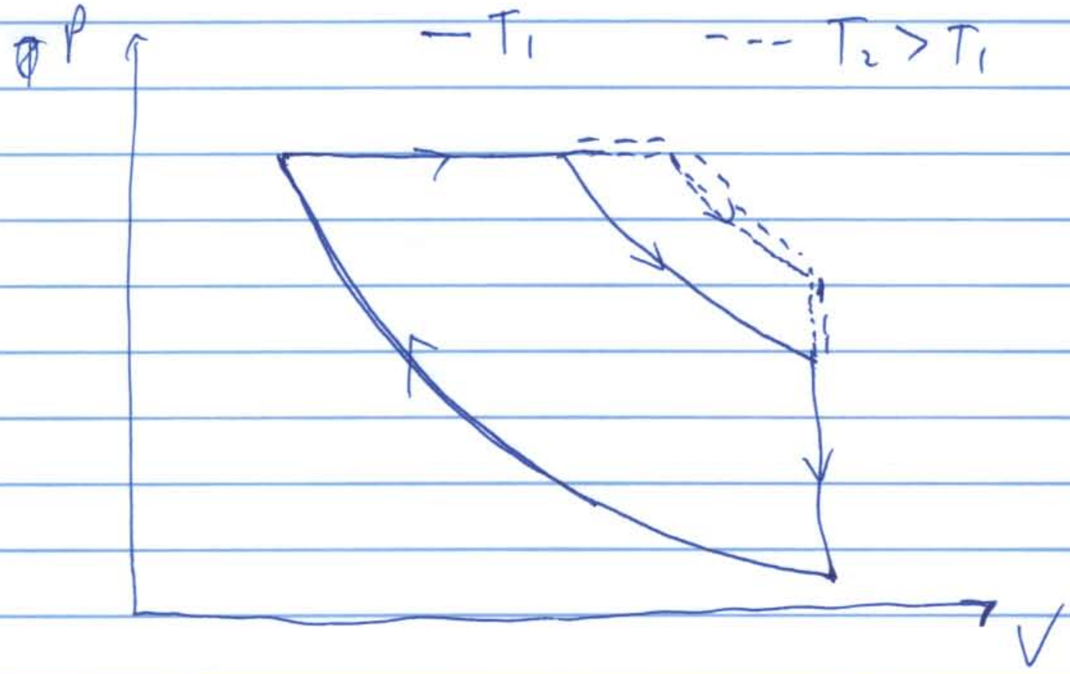
$$= 43.65\ kJ - 37.11\ kJ$$

$$\Rightarrow W_{tot} = 6.54\ kJ$$

$$\eta = \frac{W_{tot}}{q_{in}} = \frac{6.54\ kJ}{43.65\ kJ} = 0.15$$

2\.

(c) Work is area enclosed by P-V curve



If T_3 is increased keeping V_2 the same and (T_1, V_1 and P_1) the same then the area enclosed by the loop will <u>increase</u>

⇒ Work increases

3. Consider the following chemical reaction at P=1 atm:

$$A + \frac{1}{2}O_2 \rightarrow AO$$

where A is a hypothetical metal, and AO is its oxide. At STP (T=298 K, P=1 atm) the reaction is characterized by the following enthalpy change:

$$\Delta H^0_{298} = -800{,}000 \text{ J/mol}$$

The heat capacities of the reactants and products are given as follows (in units of J/mol-K):

A(s):	$C_P = 20 - 10\text{x}10^{-3}\,T$	298 K $< T <$ 1500 K
O_2(g):	$C_P = 29.96 + 4.18\text{x}10^{-3}\,T - 1.67\text{x}10^{5}\,T^{-2}$	298 K $< T <$ 3000 K
AO(α):	$C_P = 50 + 5.0\text{x}10^{-3}\,T$	298 K $< T <$ 800 K
AO(β):	$C_P = -1.0 + 40\text{x}10^{-3}\,T$	800 K $< T <$ 1500 K

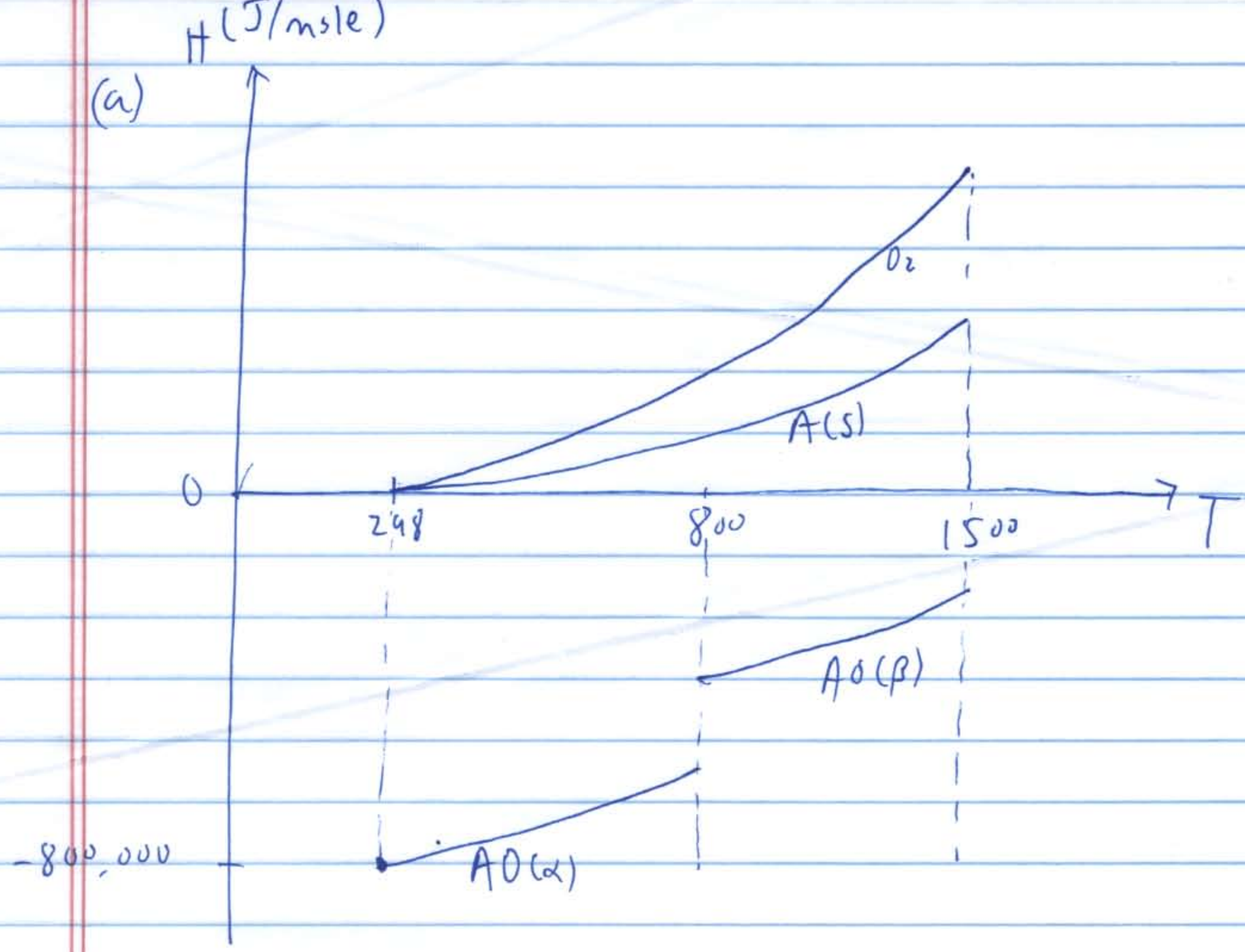
Note that AO undergoes a structural transition (α to β) at a temperature of $T^{\alpha\rightarrow\beta}_{trans} = 800$ K, and the transformation enthalpy is equal to $\Delta H^{\alpha\rightarrow\beta}_{trans} = 1000$ J/mol.

a. (10 points) Sketch the enthalpy (*H*) versus temperature (*T*) for reactants and products, from *T*=298 K to 1500 K. Label the values of *H* at *T*=298 K on your plot and clearly indicate the structural transition temperature.

b. (10 points) Compute ΔH for the reaction at *T*=1500 K.

3

(a)



(b) $$\Delta H = \Delta H^{\circ}_{298} + \int_{298}^{800} \Delta C_p(T)\, dT$$

$$+ \Delta H_{trans}^{\alpha \to \beta}$$

$$+ \int_{800}^{1500} \Delta C_p(T)\, dT$$

$= -774,296$ J/mole