Department of Mechanical Engineering
University of California at Berkeley
ME 104 Engineering Mechanics II
Spring Semester 2010

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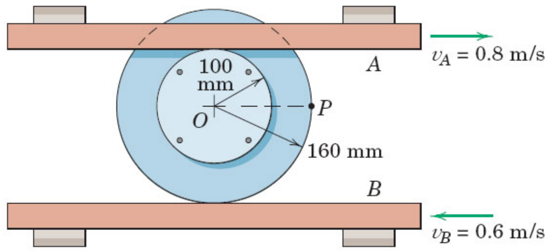
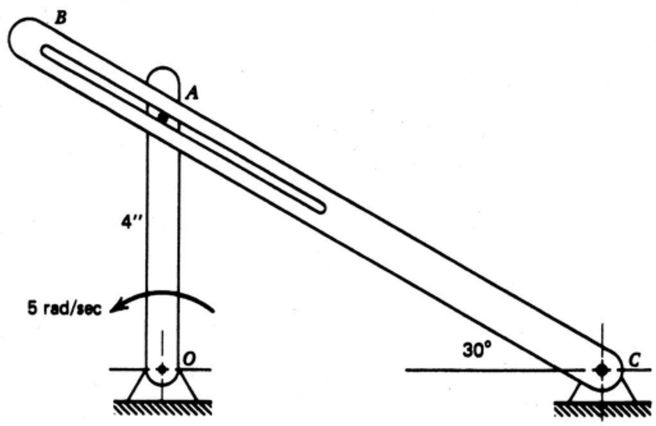
Final Examination May 11, 2010

The examination has a duration of 2 hours and 45 minutes.
Answer all questions.
All questions carry the same weight.

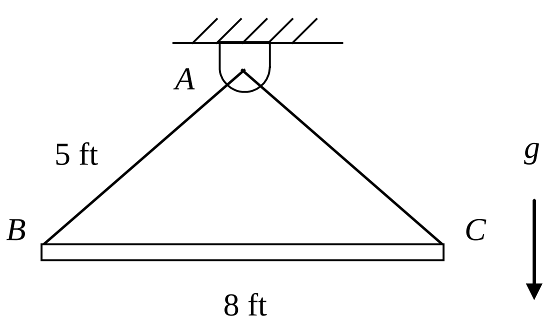
1. Each of the sliding bars A and B engages its respective rim of the two riveted wheels without slipping. If, in addition to the information shown, bar A has an acceleration of 2 m/s^2 to the right and there is no acceleration of bar B, calculate the magnitude of the acceleration of point P for the instant depicted.



2. The crank OA revolves counterclockwise with a constant angular velocity of 5 rad/s. For the position shown, determine the angular velocity and angular acceleration of the slotted link BC.

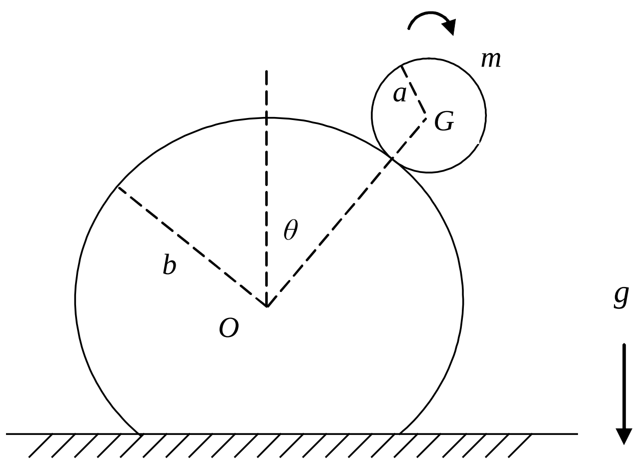


3. The slender 150-lb bar is supported by two identical cords AB and AC. If cord AC suddenly breaks, determine the initial angular acceleration of the bar and the tension in cord AB.

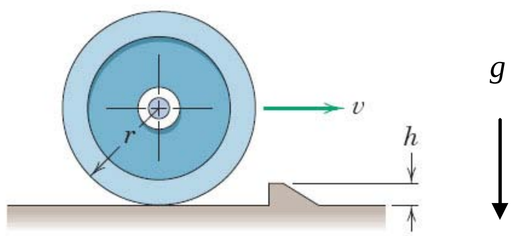


4. A sphere of mass m and radius a rests on top of a larger fixed sphere of radius b. The smaller sphere is slightly displaced so that it rolls without slipping down the larger sphere. Where will the rolling sphere leave the fixed sphere? Is there any change in the take-off position if two cylinders of radii a and b are used instead? The moment of inertia of a sphere of mass m and radius r about a diameter is $2mr^2/5$. The moment of inertia of a cylinder of mass m and

radius r about its axis is $mr^2/2$.



5. Determine the minimum velocity v which the wheel must have to just roll over the obstruction. The centroidal radius of gyration of the wheel is k, and it is assumed that the wheel does not slip. What is the value of v if the wheel is a uniform disk with mass m and radius r and $h = r/8$?



Problem 1. For *A* and *B* on the riveted wheel,

$$(a_{A/B})_t = r_{A/B}\alpha \qquad \Rightarrow \qquad 2 = (0.26)\alpha$$
$$\Rightarrow \qquad \alpha = 7.69\ \text{rad/s}^2$$
$$v_{A/B} = r_{A/B}\omega \qquad \Rightarrow \qquad 0.8 - (-0.6) = (0.26)\omega$$
$$\Rightarrow \qquad \omega = 5.38\ \text{rad/s}$$

The center *O* is in rectilinear motion along the *x*-axis and

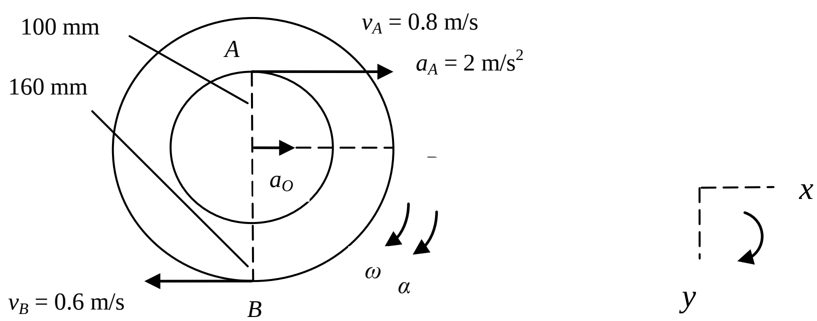
$$a_O = (0.16)\alpha = 1.23\,\text{m/s}^2$$

For *O* and *P* on the wheel,

$$\begin{aligned}
\mathbf{a}_P &= \mathbf{a}_O + (\mathbf{a}_{P/O})_n + (\mathbf{a}_{P/O})_t \\
&= 1.23\mathbf{i} - r_{P/O}\omega^2\mathbf{i} + r_{P/O}\alpha\mathbf{j} \\
&= 1.23\mathbf{i} - 0.16(5.38)^2\,\mathbf{i} + 0.16(7.69)\alpha\mathbf{j} \\
&= -3.41\mathbf{i} + 1.23\mathbf{j}
\end{aligned}$$

Thus

$$a_P = \sqrt{3.41^2 + 1.23^2} = 3.62\,\text{m/s}^2 \qquad 19.8^{\text{o}}$$



Problem 2. Attach a rotating *xy*-frame to *C* with the *y*-axis directed along *CB*. The pin *A* can only slide along the slot on link *CB*, therefore both $\mathbf{v}_{\text{rel}}$ and $\mathbf{a}_{\text{rel}}$ act along the link *CB*. Then

$$\mathbf{v}_A = \mathbf{v}_C + \boldsymbol{\omega}\times\mathbf{r}_{A/C} + \mathbf{v}_{\text{rel}} = \boldsymbol{\omega}\times\mathbf{r}_{A/C} + \mathbf{v}_{\text{rel}}$$
$$\Rightarrow \qquad -v_A\sin 30°\mathbf{i} + v_A\cos 30°\mathbf{j} = \omega_{BC}\mathbf{k}\times 8\mathbf{j} + v_{\text{rel}}\mathbf{j}$$
$$\Rightarrow \qquad -4(5)\sin 30°\mathbf{i} + 4(5)\cos 30°\mathbf{j} = -8\omega_{BC}\mathbf{i} + v_{\text{rel}}\mathbf{j}$$

Equating **i** coefficients,

$$\omega_{BC} = \frac{20\sin 30°}{8} = 1.25\text{rad/s}$$

Equating **j** coefficients,

$$v_{\text{rel}} = 20\cos 30° = 17.32\ \text{in/sec}$$

Since *AO* rotates with a constant angular velocity of 5 rad/s about fixed point *O*,

$$\mathbf{a}_A = (\mathbf{a}_{A/O})_n = -4(5)^2\cos 30°\mathbf{i} - 4(5)^2\sin 30°\mathbf{j} = -86.60\mathbf{i} - 50\mathbf{j}$$

In a similar manner,

$$\begin{aligned}
\mathbf{a}_A &= \mathbf{a}_C + \dot{\boldsymbol{\omega}}\times\mathbf{r}_{A/C} + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}_{A/C}) + 2\boldsymbol{\omega}\times\mathbf{v}_{\text{rel}} + \mathbf{a}_{\text{rel}} \\
&= \boldsymbol{\alpha}\times\mathbf{r}_{A/C} + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}_{A/C}) + 2\boldsymbol{\omega}\times v_{\text{rel}}\mathbf{j} + a_{\text{rel}}\mathbf{j} \\
&= \alpha_{BC}\mathbf{k}\times 8\mathbf{j} + 1.25\mathbf{k}\times(1.25\mathbf{k}\times 8\mathbf{j}) + 2(1.25)\mathbf{k}\times 17.32\mathbf{j} + a_{\text{rel}}\mathbf{j}
\end{aligned}$$

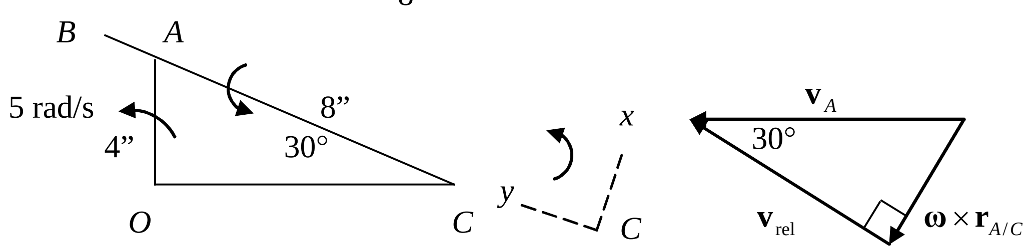
$$= -8\alpha_{BC}\mathbf{i} - 43.30\mathbf{i} - 12.5\mathbf{j} + a_{\text{rel}}\mathbf{j}$$

It follows that

$$-86.60\mathbf{i} - 50\mathbf{j} = -8\alpha_{BC}\mathbf{i} - 43.30\mathbf{i} - 12.5\mathbf{j} + a_{\text{rel}}\mathbf{j}$$

Equating **i** coefficients,

$$\alpha_{BC} = \frac{43.30}{8} = 5.41\,\text{rad/s}^2 \quad \circlearrowleft$$

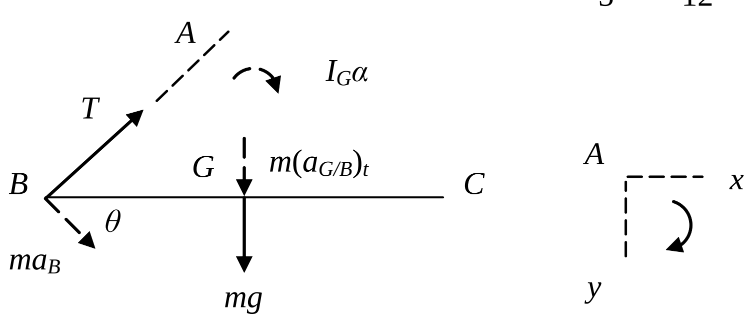


Problem 3. For the bar,

$$\sum F_x = m(a_G)_x \quad \Rightarrow \quad T\cos\theta = \frac{4}{5}T = m(a_G)_x \qquad (1)$$

$$\sum F_y = m(a_G)_y \quad \Rightarrow \quad mg - T\sin\theta = mg - \frac{3}{5}T = m(a_G)_y \qquad (2)$$

$$\sum M_G = I_G\alpha \quad \Rightarrow \quad T(4\sin\theta) = \frac{12}{5}T = \frac{1}{12}m8^2\alpha \qquad (3)$$



There are three equations containing four unknowns T, $(a_G)_x$, $(a_G)_y$ and α. From kinematics,

$$\mathbf{a}_G = \mathbf{a}_B + \mathbf{a}_{G/B}$$

Since the bar is stationary when the cord AC breaks, $\omega = 0$ and $v_B = 0$. Hence,

$$(a_{G/B})_n = 4\omega^2 = 0 \quad \Rightarrow \quad a_{G/B} = (a_{G/B})_t = 4\alpha$$

$$(a_B)_n = \frac{v_B^2}{5} = 0 \quad \Rightarrow \quad a_B = (a_B)_t$$

Thus $\mathbf{a}_{G/B}$ is perpendicular to BG and $\mathbf{a}_B$ is perpendicular to AB. Suppose $\mathbf{a}_B$ makes an angle θ with BG. Then

$$(a_G)_x = (a_B)_x + (a_{G/B})_x \quad \Rightarrow \quad (a_G)_x = a_B\cos\theta = \frac{3}{5}a_B$$

$$(a_G)_y = (a_B)_y + (a_{G/B})_y \quad \Rightarrow \quad (a_G)_y = a_B\sin\theta + 4\alpha = \frac{4}{5}a_B + 4\alpha$$

Eliminate a_B from the above two equations,

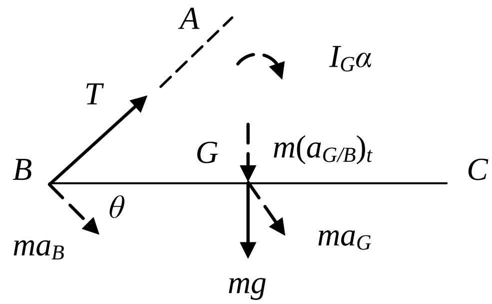
$$(a_G)_y = \frac{4}{3}(a_G)_x + 4\alpha \tag{4}$$

Solve Eqs. (1) – (4) simultaneously,

$$\alpha = \frac{27}{208} g = 4.18\,\text{rad/s}^2$$

$$T = \frac{20}{9} m\alpha = \frac{15}{52} mg = 43.27\,\text{lb}$$

In addition, $(a_G)_x = 7.43\,\text{ft/s}^2$, $(a_G)_y = 26.63$ ft/s^2, and $a_B = 12.38$ ft/s^2. As a consequence, $m\mathbf{a}_G$ has a direction as shown.



In comparison,

$$T_{st} = \frac{mg}{2\cos(90° - \theta)} = \frac{mg}{2\sin\theta} = \frac{5}{8} mg = 93.75 > T = 43.27$$

Alternative Solution

Using the moment equation about B,

$$\sum M_B = I_G\alpha \pm ma_G d = I_G\alpha + m(a_G)_y d$$

$$\Rightarrow \quad mg(4) = \frac{1}{12} m8^2 \alpha + m(a_G)_y(4) \tag{5}$$

Solve Eqs. (1), (2), (4), and (5) simultaneously,

$$\alpha = 4.18\,\text{rad/s}^2$$
$$T = 43.27\,\text{lb}$$

Problem 4. Let ω and α be angular velocity and acceleration of the rolling sphere. The position of the mass center G of the rolling sphere may be specified by the absolute coordinate θ with respect to the vertical. From kinematics, the contact point C is the instantaneous center of zero velocity and

$$\mathbf{v}_G = \mathbf{v}_C + \mathbf{v}_{G/C} = \mathbf{v}_{G/C}$$

$$\Rightarrow \quad (a+b)\dot{\theta} = a\omega$$

At any position θ before the rolling sphere leaves the fixed sphere,

$$\Delta T + \Delta V_g = 0$$

$$\Rightarrow \quad \frac{1}{2} mv_G^2 + \frac{1}{2} I_G\omega^2 - mg(a+b)(1-\cos\theta) = 0$$

$$\Rightarrow \quad \frac{1}{2} m[(a+b)\dot{\theta}]^2 + \frac{1}{2}\left(\frac{2}{5} ma^2\right)\left(\frac{(a+b)\dot{\theta}}{a}\right)^2 = mg(a+b)(1-\cos\theta)$$

$$\Rightarrow \qquad \dot{\theta}^2 = \frac{10g}{7(a+b)}(1-\cos\theta) \qquad (1)$$

For the rolling sphere,

$$\sum F_n = m(a_G)_n \qquad \Rightarrow \qquad mg\cos\theta - N = m(a+b)\dot{\theta}^2 \qquad (2)$$

Combine Eqs. (1) and (2),

$$N = \frac{1}{7}mg(17\cos\theta - 10) \qquad (3)$$

When the rolling sphere leaves the fixed sphere, $N = 0$ and

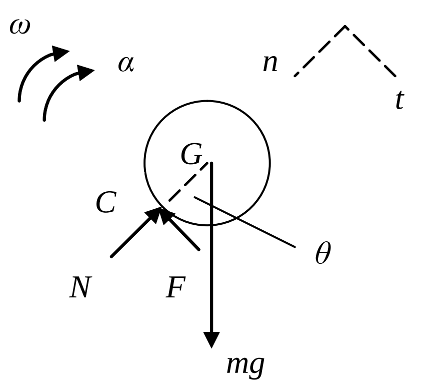
$$\theta = \cos^{-1}\frac{10}{17} = 53.97°$$

The take-off position is independent of *m*, *a*, and *b*. For two cylinders, $I_G = \frac{1}{2}ma^2$ for the rolling cylinder and the same process yields

$$\theta = \cos^{-1}\frac{4}{7} = 55.15°$$

If the rolling sphere is treated as a particle sliding on a smooth fixed sphere, the take-off position is given by

$$\theta = \cos^{-1}\frac{2}{3} = 48.19°$$



Position 1: $\theta = 0$

Position 2: $\theta > 0$

Problem 5. The motion of the wheel is divided into parts: (1) impact at *C* and (2) rolling over the obstruction afterwards. When contact occurs at *C*, the wheel pivots about the obstruction at *C* and the friction *F* between the wheel and horizontal ground vanishes. During and immediately after impact an unknown impulsive force *R* acts on the wheel over a short duration $\Delta t \approx 0$. After impact the wheel rotates about *C* with a velocity v_2 perpendicular to *GC*. Since there is no slipping,

$$v = r\omega_1$$
$$v_2 = r\omega_2$$

Just before impact,

$$(H_C)_1 = I_G\omega_1 + mv(r-h) = mk^2\frac{v}{r} + mv(r-h)$$

Just after impact,

$$(H_C)_2 = I_G\omega_2 + mv_2 r = mk^2 \frac{v_2}{r} + mv_2 r$$

Let s be the distance of C from the vertical line through G. Then

$$\int_{\Delta t} \sum M_C dt = \Delta H_C$$

$$\Rightarrow \quad -\int_{\Delta t} mgsdt = -mgs\Delta t = (H_C)_2 - (H_C)_1$$

$$\Rightarrow \quad (H_C)_1 = (H_C)_2$$

$$\Rightarrow \quad v_2 = v\left(1 - \frac{rh}{k^2 + r^2}\right) \tag{1}$$

After impact, the wheel rolls on curb point C against gravity. Thus

$$\Delta T + \Delta V_g = 0$$

For minimum v, kinetic energy after impact is totally expended in rolling over the obstruction,

$$\frac{1}{2}mv_2^2 + \frac{1}{2}I_G\omega_2^2 = mgh$$

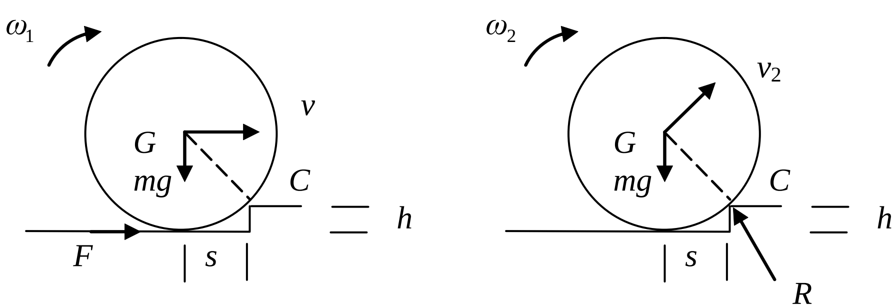
$$\Rightarrow \quad \frac{1}{2}mv_2^2 + \frac{1}{2}mk^2\frac{v_2^2}{r^2} = mgh \tag{2}$$

Combine Eqs. (1) and (2) to eliminate v_2,

$$v = \frac{r}{k^2 + r^2 - rh}\sqrt{2gh(k^2 + r^2)}$$

Suppose the wheel is a uniform disk with mass m and radius r and $h = r/8$. Since $k = r/\sqrt{2}$, the minimum velocity v which the wheel must have to just roll over the obstruction is

$$v = \frac{r}{k^2 + r^2 - rh}\sqrt{2gh(k^2 + r^2)} = \frac{\sqrt{24gr}}{11} = 0.4454\sqrt{gr}$$



Just before impact at $t = 0$ Just after impact at $t = \Delta t$