Physics 7B Midterm 1

Problem 1

$$F(\vec{v}) = \frac{1}{Z} e^{-m(\vec{v}-\vec{u})^2/2kT}$$

a)1 $\langle \vec{v} \rangle = ?$ A probability distribution function for velocity v is averaged at 0. In this case, we make a change of variable to $\vec{\omega} = \vec{v} - \vec{u}$. The probability distribution is then written in the more familiar way: $F(\vec{v}) = \frac{1}{Z} e^{-m\vec{\omega}^2/2kT}$

Since $\vec{\omega} = \vec{v} - \vec{u}$, the velocity average is now shifted by $\vec{u}$. Therefore $\langle \vec{v} \rangle = \vec{u}$.

a)2 $\langle v^2 \rangle$ From the textbook, we have $\langle v^2 \rangle = \frac{2}{m}$ [redacted] $(3/2)$ [redacted] kT. But since we have velocity $= \vec{\omega} = \vec{v} - \vec{u}$, we have

$$\langle v^2 \rangle = \frac{3\ \text{[redacted]}\ kT}{m} + \vec{u}^2$$

a)3 $V_{rms} = ?$ From the textbook, we have

$$V_{rms} = \sqrt{\langle v^2 \rangle}$$

Plugging in, we have

$$V_{rms} = \sqrt{\left(\frac{3\ \text{[redacted]}\ kT}{m}\right) + |\vec{u}|^2}$$

b) F(v) is maximized when $\vec{v} = \vec{u}$:

$$F(\vec{v}) = \frac{1}{Z} e^{-m(0)^2/2kT} = \frac{1}{Z}$$

peak velocity: $\vec{v} = \vec{u}$

Note:

To see why $\langle v^2 \rangle = \frac{3kT}{m} + u^2$ we note that for $\vec{w} = \vec{V} - \vec{u}$ the distribution is the standard Maxwell distribution so

$$\langle w^2 \rangle = \frac{3kT}{m}$$

but

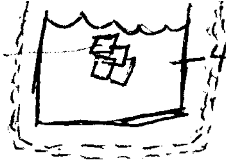
$$\langle w^2 \rangle = \langle (\vec{v} - \vec{u})^2 \rangle = \langle v^2 \rangle - 2 \langle \vec{u} \cdot \vec{v} \rangle + \langle u^2 \rangle$$
$$= \langle v^2 \rangle - 2 \vec{u} \cdot \langle \vec{v} \rangle + u^2$$

since $\langle \vec{v} \rangle = \vec{u}$, this gives

$$\langle w^2 \rangle = \langle v^2 \rangle - u^2 = \frac{3kT}{m}$$

$$\Rightarrow \quad \langle v^2 \rangle = \frac{3kT}{m} + u^2$$

#2)

a) $0^\circ C$  $49^\circ C$ $\quad \Delta Q = 0$

* the ice will melt & then heat up from $0^\circ C$ to $6^\circ C$
* the water will cool from $49^\circ C$ to $6^\circ C$

$$\Delta Q_{\substack{ice \\ melt}} + \Delta Q_{\substack{water \\ cool}} + \Delta Q_{\substack{ice\,water \\ heat}} = 0$$

$$\underbrace{\cancel{(.1\,kg)}\,L_{fusion} + \overset{2}{\cancel{(.2\,kg)}}\,c_{H_2O}(6^\circ C - 49^\circ C) + \cancel{(.1\,kg)}\,c_{H_2O}(6^\circ C - 0^\circ C)}_{=0}$$

$$\Rightarrow L_{fusion} + 2\left(1\,\frac{J}{K\cdot kg}\right)(-43\,K) + 1\left(1\,\frac{J}{K\,kg}\right)(6\,K) = 0$$

$$L_{fusion} = 86\,J - 6\,J = \boxed{80\,J/kg}$$

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b) $\Delta S = \Delta S_{I} + \Delta S_{II} + \Delta S_{III}$

* 3 processes are happening
  - I → the ice melts in the water
  - II → the newly made water is heated
  - III → the original water is cooled

$$\Delta S_{I} = \frac{m_{ice}L_{fusion}}{T} = \frac{(.1\,kg)(80\,J/kg)}{273\,K} \approx \boxed{.029\,\frac{J}{K}}$$

$$\Delta S_{II} = m_{H_2O}\,c\int_{273}^{279}\frac{dT}{T} = (.1\,kg)(1\,J/K\,kg)\ln\left(\frac{279}{273}\right) = \boxed{.00217\,J/K}$$

$$\Delta S_{III} = m_{H_2O}\,c\int_{322}^{279}\frac{dT}{T} = (.2\,kg)(1\,J/K\,kg)\ln\left(\frac{279}{322}\right) = \boxed{-.0286\,\frac{J}{K}}$$

$$\Delta S = .029\,J/K + .00217\,J/K - .0286\,J/K = \boxed{.00257\,J/K}$$

$\parallel$

total ΔS

Question Three

$W_{isothermal} = P_0V_0 \ln\left(\frac{V_f}{V_0}\right)$ $\quad W_{isochoric} = 0$ $\quad P_0 = \frac{NKT_0}{V_0}$

$Q_{isothermal} = P_0V_0 \ln\left(\frac{V_f}{V_0}\right)$ $\quad Q_{isochoric} = \frac{d}{2}V_0(P_f - P_0)$

diatomic $\rightarrow d = 5$

Plug in the above equations for each point on the graph,

I) $P_1 = \frac{NKT_1}{V_1}$ $\quad T_1 = T_H$

$\boxed{W_I = T_H NK \ln\left(\frac{V_2}{V_1}\right)}$

$\boxed{Q_I = W_I}$

II) $\Delta V = 0$

$\boxed{W_{II} = 0}$

$Q_{II} = \frac{5}{2}V_2(P_3 - P_2)$

$\boxed{Q_{II} = \frac{5}{2}NK(T_C - T_H)}$

III) $\boxed{W_{III} = NKT_C \ln\left(\frac{V_1}{V_2}\right)}$

$\boxed{W_{III} = Q_{III}}$

IV) $\boxed{W_{IV} = 0}$

$\boxed{Q_{IV} = \frac{5}{2}NK(T_H - T_C)}$

note: $NK = nR$

$T_I = T_{II} = T_H$

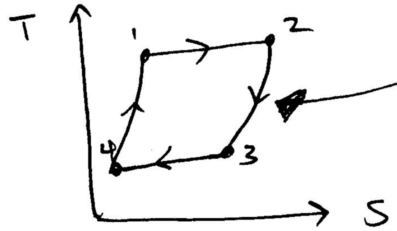
$T_{III} = T_{IV} = T_C$

$V_I = V_{IV}$

$V_{II} = V_{III}$

Question Three

b) This is a Stirling engine



The isochoric legs go as the log

for the isochoric sections

$$dQ = \frac{5}{2} N k \, dT$$

$$\int dS = \int \frac{dQ}{T} = \frac{5}{2} N k \int \frac{dT}{T} = \frac{5}{2} N k \ln(T) + \text{const.}$$

c) $$e = \frac{W}{Q_H} = \frac{W_{I} + W_{III}}{Q_{I} + Q_{IV}}$$

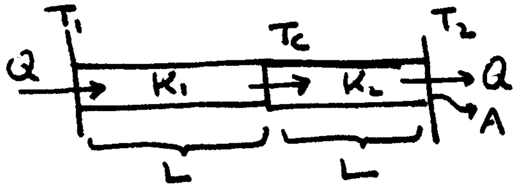
$$= \frac{N\cancel{k} \ln(V_2/V_1)(T_H - T_C)}{T_H \cancel{Nk} \ln(V_2/V_1) + \frac{5}{2} \cancel{Nk}(T_H - T_C)} \times 100\%$$

$$= \frac{\ln(5)(400-300)}{400 \ln(5) + \frac{5}{2}(400-300)} \times 100\%$$

$e = 18\%$

4 (a)

SERIES:



$T_1 > T_2$

$\longrightarrow x$

- In steady state . Temperature only depends on location 'x' not time
- Heat flow $\frac{dQ}{dt}$ constant throughout.

$$\frac{dQ}{dt} = \text{CONST} = \frac{K_1 A}{L}(T_1 - T_c) = \frac{K_2 A}{L}(T_c - T_2)$$

$$\Rightarrow K_1 T_1 - K_1 T_c = K_2 T_c - K_2 T_2 \Rightarrow \boxed{T_c = \frac{K_1}{K_1+K_2} \cdot T_1 + \frac{K_2}{K_1+K_2} \cdot T_2}$$

$\hookrightarrow$ Temp. at the interface

Plug T_c into $\frac{dQ}{dt}$ above, say 1^{st} one:

$$\frac{dQ}{dt} = \frac{K_1 A}{L}\left(T_1 - \frac{K_1 T_1}{K_1+K_2} - \frac{K_2 T_2}{K_1+K_2}\right) = \frac{K_1 A}{L}\left(\cancel{K_1 T_1} + K_2 T_1 - \cancel{K_1 T_1} - K_2 T_2\right) \cdot \frac{1}{K_1+K_2}$$

$$\Rightarrow \frac{dQ}{dt} = \frac{K_1 K_2}{K_1+K_2} \cdot \frac{A}{L}(T_1 - T_2) = \frac{2K_1 K_2}{K_1+K_2} \cdot \frac{A}{2L} \cdot (T_1 - T_2)$$

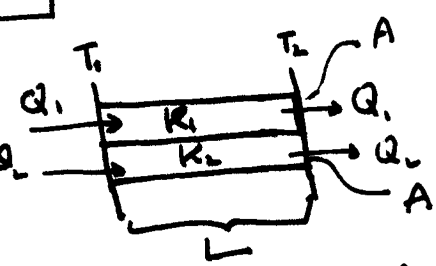
$$\boxed{\frac{dQ}{dt} = \underbrace{\frac{2K_1 K_2}{K_1+K_2}}_{\equiv K_{eff}} \cdot \frac{A}{2L}(T_1 - T_2)}$$

$\hookrightarrow$ single rod with length $2L$, crossection A

$$\Rightarrow \boxed{K_{eff} = \frac{2K_1 K_2}{K_1+K_2}}$$

Check: $K_1 = K_2 \Rightarrow K_{eff} = K$ ✓

(b) Parallel:



$T_1 > T_2$

In steady state heat flow through each constant

$$\frac{dQ}{dt} = \frac{dQ_1}{dt} + \frac{dQ_2}{dt} = \frac{K_1 A}{L}(T_1 - T_2) + \frac{K_2 A}{L}(T_1 - T_2)$$

$$\frac{dQ}{dt} = (K_1 + K_2)\frac{A}{L}(T_1 - T_2) \Rightarrow \boxed{\frac{dQ}{dt} = \underbrace{\left(\frac{K_1+K_2}{2}\right)}_{\equiv K_{eff}}\frac{2A}{L} \cdot (T_1 - T_2)}$$

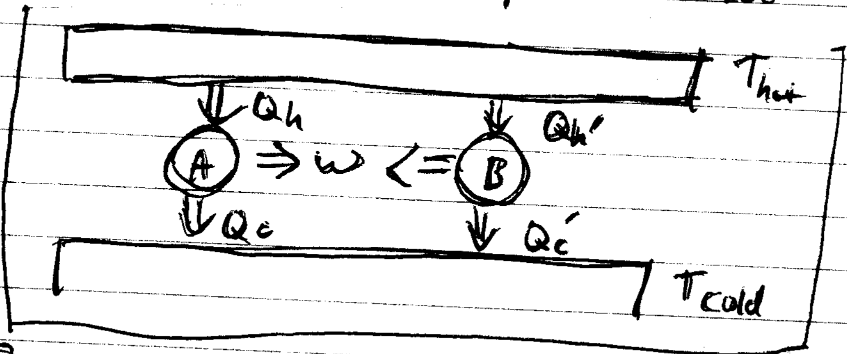
$\hookrightarrow$ single rod of $2A$ and L

$$\boxed{K_{eff} = \frac{K_1+K_2}{2}}$$

Check: $K_1 = K_2 \Rightarrow K_{eff} = K$ ✓

Suppose there exists a hypothetical engine (B) w/ greater efficiency than a Carnot Engine (A)

We adjust the cycle of the Carnot engine such that it's work output $W_{carnot} = W_{hypothetical} = W$.

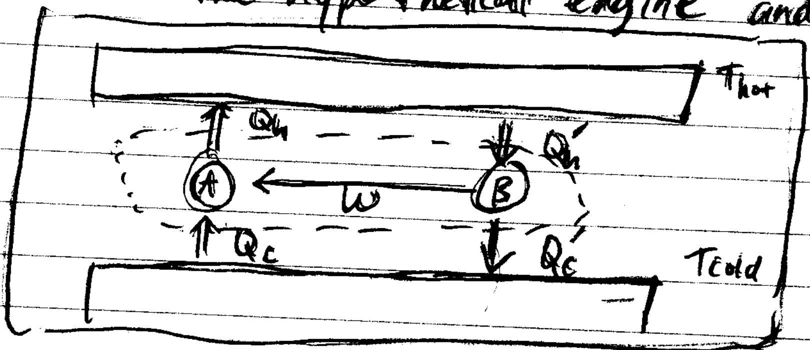


This means

$Q_h' - Q_c' = W$

$Q_h - Q_c = W$

Since the Carnot engine is reversible, we can turn it into a refrigerator that takes in work W from the hypothetical engine and energy Q_c from the cold reservoir and the outputs energy Q_h into the hot reservoir.



Let's look at what this means:

$$\frac{W}{Q_h'} > \frac{W}{Q_h} \Rightarrow \boxed{Q_h > Q_h'}$$

also

$$\left.\begin{matrix} Q_c + W = Q_h \\ -Q_c' - W = -Q_h' \end{matrix}\right\} \Rightarrow \boxed{Q_c - Q_c' = Q_h - Q_h' > 0}$$

Look at the net effect of the 2-engine contraption we made. It takes a positive amount of heat ($Q_c - Q_c'$) from the cold reservoir & moves it to the hot reservoir with no work! This violates the 2nd law!

This cannot be!!