1. Dimensional Analysis & Model Testing

a) $d = d(D, V, \rho, \eta, \sigma)$

Variable	Dimensions
d	L
D	L
V	L/T
ρ	M/L^3
η	$M/L\cdot T$
σ	$\frac{F}{L} = \frac{M\cdot L}{T^2 \cdot L} = \frac{M}{T^2}$

6 Variables

3 Dimensions

$\Rightarrow 6-3=3$ Dimensionless Groups (N_1, N_2, N_3)

pick core group : ρ, V, D (as in all examples in lecture)

$N_1 = \frac{d}{D}$ (by inspection)

$N_2 = \frac{\rho V D}{\eta}$ (from experience!)

$N_3 = \rho^a V^b D^c \sigma = M^0 L^0 T^0$

$$\left(\frac{M}{L^3}\right)^a \left(\frac{L}{T}\right)^b (L)^c \left(\frac{M}{T^2}\right) = M^0 L^0 T^0$$

M: $a + 1 = 0 \Rightarrow a = -1$

T: $-b - 2 = 0 \Rightarrow b = -2$

L: $-3a + b + c = 0 \Rightarrow 3 - 2 + c = 0 \Rightarrow c = -1$

$$N_3 = \frac{\sigma}{\rho V^2 D}$$

$$\boxed{\frac{d}{D} = \frac{d}{D}\left(\frac{\rho V D}{\eta}, \frac{\sigma}{\rho V^2 D}\right)}$$

b).

$$\frac{d_M}{D_M} = \frac{d_P}{D_P} \quad \text{when}$$

$$\frac{\rho_M V_M D_M}{\eta_M} = \frac{\rho_P V_P D_P}{\eta_P} \quad \text{and} \quad \frac{\sigma_M}{\rho_M V_M^2 D_M} = \frac{\sigma_P}{\rho_P V_P^2 D_P}$$

i.e. $N_{1,M} = N_{1,P}$, $N_{2,M} = N_{2,P}$ & $N_{3,M} = N_{3,P}$.

c). $D_M = 1$ mm and $D_P = 6$ mm

If same fluid used, then $\sigma_M = \sigma_P$, $\rho_M = \rho_P$, & $\eta_M = \eta_P$

so

$$\frac{\rho V_M D_M}{\eta} = \frac{\rho V_P D_P}{\eta}$$

$$\Rightarrow \boxed{\frac{V_M}{V_P} = \frac{D_P}{D_M} = \frac{6}{1} = 6} \quad \text{(A)}$$

But we also need

$$\frac{\sigma}{\rho V_M^2 D_M} = \frac{\sigma}{\rho V_P^2 D_P}$$

$$\frac{V_P^2}{V_M^2} = \frac{D_M}{D_P} \quad \text{or} \quad \boxed{\frac{V_M}{V_P} = \sqrt{\frac{D_P}{D_M}} = \sqrt{6}} \quad \text{(B)}$$

$\Rightarrow$ Thus, comparing (A) & (B), we cannot satisfy criteria if same fluid is used in model & prototype.

2. Note that $\Delta p_A = \Delta p_B$

We are given $Q_B \Rightarrow$ can calculate V_B

$$V_B = \frac{Q_B}{A_B} = \frac{4Q_B}{\pi D_B^2} = \frac{4\left(7.07 \times 10^{-5}\ \frac{m^3}{s}\right)}{\pi (0.05\,m)^2} = 0.036\ \frac{m}{s}$$

$$Re_B = \frac{\rho V_B D_B}{\eta} = \frac{\left(10^3\ \frac{kg}{m^3}\right)\left(0.036\ \frac{m}{s}\right)(0.05\,m)}{10^{-3}\ Pa \cdot s} = 1800$$

Flow is Laminar $\Rightarrow$ Calc. Δp_B from Hagen-Poiseuille Eqn. (or from $f = \frac{16}{Re}$)

H-P :
$$\Delta p_B = \frac{128}{\pi} Q_B L \eta \frac{1}{D^4}$$

$$= \frac{128}{\pi}\left(7.07 \times 10^{-5}\ \frac{m^3}{s}\right)(1\ m)\left(10^{-3}\ Pa\text{-}s\right)\frac{1}{(0.05\,m)^4}$$

$$= 0.461\ Pa$$

(check: $f = \frac{16}{Re} = \frac{16}{1800} = \frac{\Delta p D}{2 \rho V^2 L}$

$$\Delta p_B = \frac{2(16)(10^3)(0.036)^2(1)}{(1800)(0.05)} = 0.461\ \checkmark\ )$$

Since $\Delta p_A = \Delta p_B$, we can now calc. Q_A from either

a) Hagen-Poiseuille equation if flow in pipe A is laminar or

b) Blasius eqn if flow in pipe A is turbulent.

Since pipe A is larger, guess that flow is turbulent

$$\Rightarrow\ Q_A = 2.26\left(\frac{\Delta p_A}{L}\right)^{4/7}\left(\rho^3 \eta\right)^{-1/7} D_A^{19/7}$$ (Eqn. 3.11, Blasius rearranged).

$$= 2.26\left(\frac{0.461}{1}\right)^{4/7}\left((10^3)^3(10^{-3})\right)^{-1/7}(0.15)^{19/7}$$

$$= 1.17 \times 10^{-3}\ m^3/s$$

check assumption that $Re_A > 2100$:

$$V_A = \frac{Q_A}{A_A} = \frac{4Q_A}{\pi D_A^2} = \frac{4(1.17\times10^{-3}\,m^3/s)}{\pi\,(0.15\,m)^2} = 6.625\times10^{-2}\,\frac{m}{s}$$

$$Re_A = \frac{\rho V_A D_A}{\eta} = \frac{(10^3)(6.625\times10^{-2})(0.15)}{10^{-3}} = 9938 \checkmark$$

turbulent!

$\Rightarrow$ From a mass balance,

$$Q_{Total} = Q_A + Q_B$$

$$\Rightarrow \frac{Q_B}{Q_{Total}} = \frac{Q_B}{Q_A+Q_B} = \frac{7.07\times10^{-5}}{7.07\times10^{-5}+1.17\times10^{-3}}$$

$$= \boxed{\underline{5.7\,\%} \text{ of flow goes thru pipe B.}}$$

If you assumed flow in A was laminar, since $\Delta p_A = \Delta p_B$, from Hagen-Poiseuille:

$$\frac{128}{\pi} Q_B L \eta \frac{1}{D_B^4} = \frac{128}{\pi} Q_A L \eta \frac{1}{D_A^4}$$

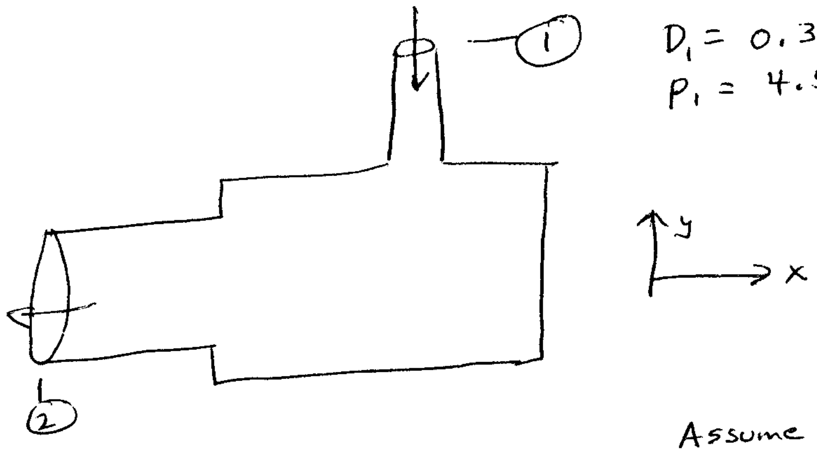
$$\Rightarrow \frac{Q_A}{Q_B} = \frac{D_A^4}{D_B^4} = 3^4 = 81$$

$$\Rightarrow Q_A = 81\,Q_B = 5.727\times10^{-3}\,\frac{m^3}{s} \quad \& \quad \frac{Q_B}{Q_T} = \frac{1}{82} = \underline{1.2\,\%}$$

$$V_A = \frac{Q_A}{A_A} = 3.24\times10^{-1}\,m/s$$

$$\Rightarrow Re_A = \frac{\rho V_A D_A}{\eta} = 4.86\times10^4\,!$$ $\leftarrow$ inconsistent w/ laminar assumption

3.



$D_1 = 0.30$ m
$P_1 = 4.5 \times 10^5$ Pa

$D_2 = 0.60$ m
$P_2 = 2.2 \times 10^5$ Pa

Assume flow is turbulent at ① & ②

Assume steady-state

$$W = 300 \frac{kg}{s} = \rho V_1 A_1 = \rho V_2 A_2$$

⇒ Want F_x, F_y so we need a momentum balance.

$$0 = w(\beta_1 \langle \underline{V} \rangle_1 - \beta_2 \langle \underline{V} \rangle_2) + P_1 \underline{A}_1 - P_2 \underline{A}_2 - \underline{F} + \int \rho A dz \, \underline{g}$$

Flow is in horizontal plane so $g_x = g_y = 0$

Calculate V_1 & V_2:

$$V_1 = \frac{w}{\rho A_1} = \frac{4w}{\rho \pi D_1^2} = \frac{4(300 \frac{kg}{s})}{(10^3 \frac{kg}{m^3})(\pi)(0.30 m)^2} = 4.24 \frac{m}{s}$$

$$V_2 = \frac{w}{\rho A_2} = \frac{4w}{\rho \pi D_2^2} = 1.06 \frac{m}{s}$$

x-comp of momentum:

$$0 = w(-(-V_2)) - P_2(-A_2) - F_x$$

$$F_x = w V_2 + P_2 A_2 = (300 \frac{kg}{s})(1.06 \frac{m}{s}) + (2.2 \times 10^5 \text{ Pa}) \pi \frac{(0.60)^2}{4}$$

$$= \underline{6.25 \times 10^4 \text{ N}}$$ (Force by fluid is in + x direction)

y-comp. of momentum balance is

$$0 = w(-V_1) + p_1(-A_1) - F_y$$

$$F_y = -wV_1 - p_1 A_1$$

$$= -\left[\left(300\ \frac{kg}{s}\right)\left(4.24\ \frac{m}{s}\right) + \left(4.5 \times 10^5\ Pa\right)\pi\,\frac{(0.30\ m)^2}{4}\right]$$

$$= \underline{-3.31 \times 10^4\ N} \quad \text{in negative } y \text{ direction}$$