1. Let's first see if she can clear the intersection by accelerating at the largest rate her car is capable of.

The problem says that the car can at best accelerate from 45 km/h to 60 km/h in 6 seconds. Noting that 45 km/h = 12.5 m/s and 60 km/h = 16.7 m/s,

$$a_{max} = \frac{V_f - V_o}{\Delta t} = \frac{16.7 - 12.5}{6} = .7 \text{ m/s}^2$$

The distance she could cover at this acceleration is

$$d = \frac{1}{2}at^2 + V_o t = \frac{1}{2}(.7)^2(2)^2 + (12.5)(2) = 26.4 \text{ m}$$

which is less than the 15 + 28 meters she needs to travel. So she should not accelerated.

Let's also see if she can decelerate in time. Using

$$V_f^2 = V_o^2 + 2ad \Rightarrow 0 = (12.5)^2 - 2(5.8)d$$

$$\Rightarrow d = 13.5 \text{ m}$$

So she would comfortably stop without running into the intersection.

2a) $$\left.\begin{aligned} x &= V_{ox}\, t \\ x &= V_o \cos(\theta+\alpha)\, t \end{aligned}\right\} +1$$

$$\left.\begin{aligned} y &= V_{oy}\, t - \tfrac{1}{2} g t^2 \\ y &= V_o \sin(\theta+\alpha)\, t - \tfrac{1}{2} g t^2 \end{aligned}\right\} +1$$

write $$\left.\begin{aligned} x &= d\cos\theta \\ y &= d\sin\theta \end{aligned}\right\}$$ condition when the object lands on the inclined plane } +2

① $$d\cos\theta = V_o \cos(\theta+\alpha)\, t \implies t = \frac{d\cos\theta}{V_o\cos(\theta+\alpha)}$$

② $$d\sin\theta = V_o\sin(\theta+\alpha)\, t - \frac{1}{2} g t^2$$

$$d\sin\theta = V_o\sin(\theta+\alpha)\left[\frac{d\cos\theta}{V_o\cos(\theta+\alpha)}\right] - \frac{1}{2} g\left[\frac{d\cos\theta}{V_o\cos(\theta+\alpha)}\right]^2$$

$$d\sin\theta = d\cos\theta\tan(\theta+\alpha) - \frac{g}{2V_o^2}\,\frac{d^2\cos^2\theta}{\cos^2(\theta+\alpha)}$$

$$\frac{g}{2V_o^2}\,\frac{\cos^2\theta}{\cos^2(\theta+\alpha)}\, d = \cos\theta\tan(\theta+\alpha) - \sin\theta$$

$$d = \frac{2V_o^2}{g}\left[\frac{\tan(\theta+\alpha) - \tan\theta}{\cos\theta}\right]\cos^2(\theta+\alpha)$$

$$\tan(\theta+\alpha) - \tan\theta = \frac{\sin(\theta+\alpha)}{\cos(\theta+\alpha)} - \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\sin\theta\cos\alpha + \cos\theta\sin\alpha}{\cos\theta\cos\alpha - \sin\theta\sin\alpha} - \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\sin\theta\cos\theta\cos\alpha + \cos^2\theta\sin\alpha - \sin\theta\cos\theta\cos\alpha + \sin^2\theta\sin\alpha}{\cos(\theta+\alpha)\cos\theta}$$

$$= \frac{\sin\alpha}{\cos(\theta+\alpha)\cos\theta}$$

$$\therefore\ d = \frac{2v_0^2}{g}\,\frac{\sin\alpha\cos(\theta+\alpha)}{\cos\theta} \qquad (3)$$

Solve for d +3

maximize d w.r.t. α

$$\frac{\partial d}{\partial\alpha} = 0$$

+1

$$\frac{\partial}{\partial\alpha}\left(\sin\alpha\cos(\theta+\alpha)\right) = 0$$

$$\cos\alpha\cos(\theta+\alpha) - \sin\alpha\sin(\theta+\alpha) = 0$$

$$1 = \tan\alpha\tan(\theta+\alpha)$$

$$\tan(\theta+\alpha) = \cot(\alpha)$$

But $\cot\alpha = \tan(90° - \alpha)$

$$\theta + \alpha = 90° - \alpha$$

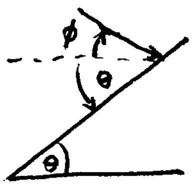
$$\boxed{\alpha = \frac{90° - \theta}{2}}$$

Solve for α +2

Plug in $\theta = 30°$ $\Rightarrow$ $\boxed{\alpha = 30°}$

Plug in # +1

2b.) angle landed = angle throwing back down + 1



$$\beta = \theta + \phi$$

$$\phi = \tan^{-1}\left(\frac{-V_y}{V_x}\right)$$ at the time the object lands + 2

find t = time that the object lands + 1

from ③ $d\cos\theta = \frac{2V_0^2}{g} \sin\alpha \cos(\theta + \alpha)$

from ① $d\cos\theta = V_0 \cos(\theta + \alpha) t$

$$1 = \frac{2V_0}{g} \frac{\sin\alpha}{t}$$

$$\therefore t = \frac{2V_0}{g} \sin\alpha$$ Solve for t + 1

$V_x = V_{0x} = V_0 \cos(\theta + \alpha)$ V_x + 1

$V_y = V_{0y} - gt = V_0 \sin(\theta + \alpha) - g\left(\frac{2V_0}{g}\sin\alpha\right)$

$V_y = V_0 \left[\sin(\theta + \alpha) - 2\sin\alpha\right]$ V_y + 1

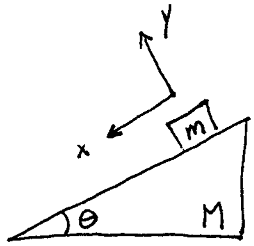
$$\boxed{\beta = \theta + \phi = \theta + \tan^{-1}\left[\frac{2\sin\alpha - \sin(\theta + \alpha)}{\cos(\theta + \alpha)}\right]}$$ + 1

Plug in $\theta = 30°$
$\alpha = 30°$

$$\boxed{\beta = 45°}$$ + 1

Yildiz
Midterm 1

a)

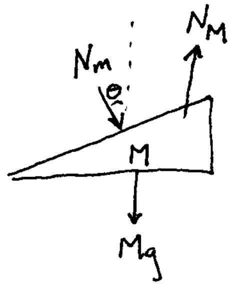


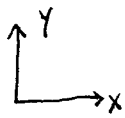
$$\sum_m F_x = mg\sin\theta = m a_{mx}$$

$$\sum_m F_y = N_m - mg\cos\theta = 0$$

$$\Longrightarrow \boxed{a_{mx} = g\sin\theta}$$

b)





① $\sum_m F_x = -N_m \sin\theta = m a_{mx}$

② $\sum_m F_y = N_m\cos\theta - mg = m a_{my}$

③ $\sum_M F_x = N_m \sin\theta = M a_{Mx}$

④ $\sum_M F_y = N_M - N_m\cos\theta - Mg = 0$

Note: $\boxed{\dfrac{a_{my}}{a_{mx} - a_{Mx}} = \tan\theta}$ (block stays on the incline)

$$\Rightarrow a_{my} = \tan\theta\,(a_{mx} - a_{Mx})$$

Now: ③ $\Rightarrow N_m = \dfrac{M a_{Mx}}{\sin\theta}$ ① $\Rightarrow N_m = \dfrac{-m a_{mx}}{\sin\theta}$

$$\Longrightarrow a_{mx} = -\frac{M}{m} a_{Mx}$$

(a) $\Rightarrow \left(\frac{M a_{Mx}}{\sin\theta}\right)\cos\theta - mg = \tan\theta\left(-\frac{M}{m} a_{Mx} - a_{Mx}\right)$

$$\Rightarrow M a_{Mx} \cot\theta + \left(\frac{M}{m} + 1\right)\tan\theta\, a_{Mx} = mg$$

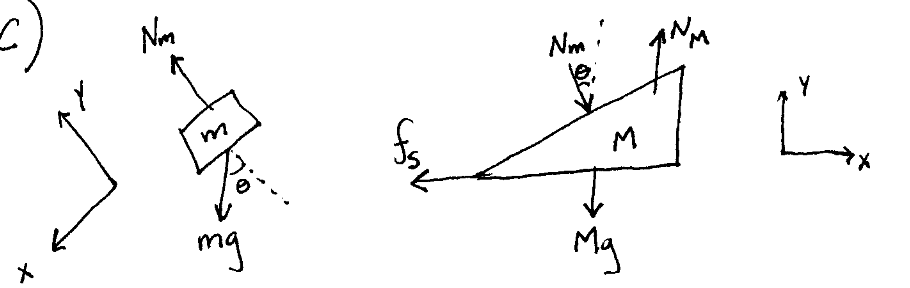
$$\Rightarrow \boxed{a_{Mx} = \frac{mg}{M\cot\theta + \left(\frac{M}{m}+1\right)\tan\theta}}$$

$$\boxed{a_{mx} = -\frac{M}{m} a_{Mx} = \frac{-Mg}{M\cot\theta + \left(\frac{M}{m}+1\right)\tan\theta}}$$

$$\boxed{a_{my} = \frac{-(M+m)\,g\,\tan\theta}{M\cot\theta + \left(\frac{M}{m}+1\right)\tan\theta}}$$

$$\boxed{\vec{a}_m = a_{mx}\,\hat{x} + a_{my}\,\hat{y}}$$

c)



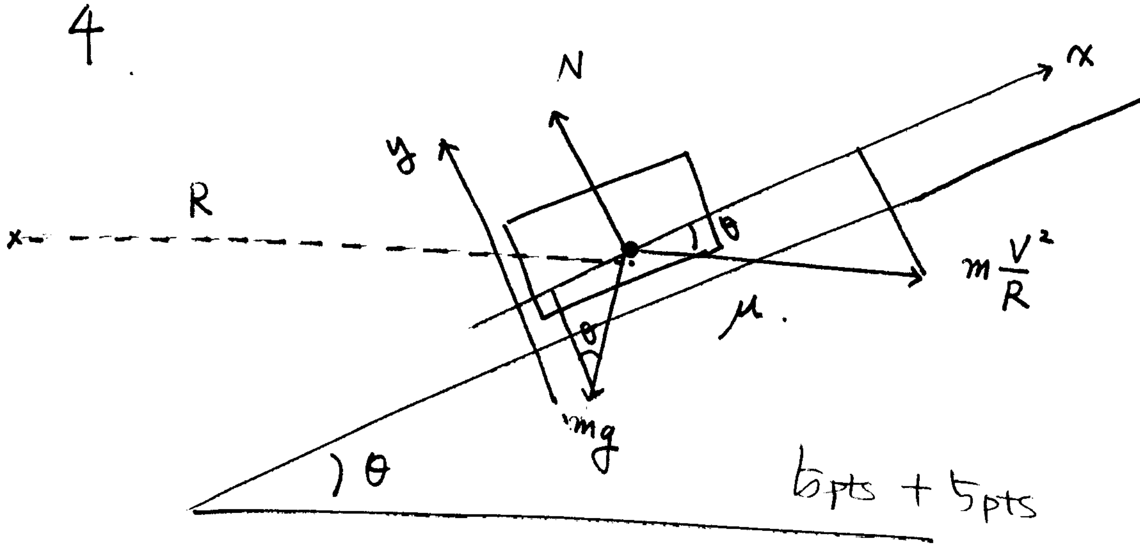
$$\sum_m F_y = N_m - mg\cos\theta = 0 \quad \Rightarrow \quad N_m = mg\cos\theta$$

$$\sum_M F_y = N_M - N_m\cos\theta - Mg = 0 \quad \Rightarrow \quad N_M = Mg + mg\cos^2\theta$$

$$\sum_M F_x = N_m\sin\theta - \mu_s N_M = 0 \quad \Rightarrow \quad mg\cos\theta\sin\theta = \mu_s\left(Mg + mg\cos^2\theta\right)$$

$$\Rightarrow \boxed{\mu_s = \frac{mg\cos\theta\sin\theta}{(Mg + mg\cos^2\theta)}}$$

4.



6pts + 5pts

$$N = mg\cos\theta + m\,V^2/R\,\sin\theta.$$
$$F_x = m\,V^2/R\,\cos\theta - mg\sin\theta \pm f$$

From $\sum_i F_i = 0$.

5pts + 5pts

f : friction force and $\pm$ depends on the direction of motions along $\hat{x}$.

and $f = \mu N$

not to skid along $\hat{x} \longrightarrow F_x = 0$.

a) $$m\,V^2/R\,\cos\theta - mg\sin\theta = \mu\,(mg\cos\theta + m\,V^2/R\,\sin\theta).$$

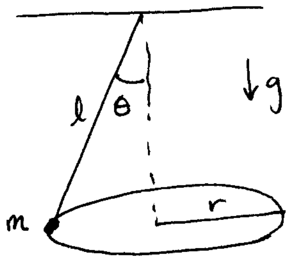
then $$m\,V^2/R\,(\cos\theta - \mu\sin\theta) = mg\,(\sin\theta + \mu\cos\theta).$$

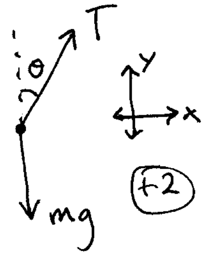
thus $$V = \sqrt{Rg\,\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}}.$$ largest speed

b) $$mv^2/R\,\cos\theta - mg\sin\theta = -\mu\,(mg\cos\theta + m\,V^2/R\,\sin\theta)$$

thus $$V = \sqrt{Rg\,\frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta}}$$ smallest speed.

5.) a.)





(+2)

$$T\cos\theta - mg = 0 \quad (+2)$$

$$T\sin\theta = m\frac{v^2}{r} \quad (+2)$$

$$r = \ell\sin\theta \quad (+1)$$

$$T = \frac{mg}{\cos\theta}$$

$$mg\tan\theta = m\frac{v^2}{r}$$

$$v = \sqrt{gr\tan\theta}$$

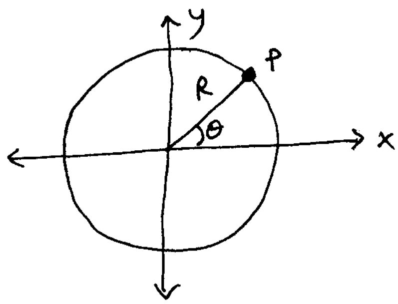
$$v = \sqrt{g\ell\frac{\sin^2\theta}{\cos\theta}} \quad (+2)$$

$$\omega = \frac{2\pi}{T} = \frac{v}{r} = \frac{\sqrt{g\ell\frac{\sin^2\theta}{\cos\theta}}}{\ell\sin\theta} \quad (+1)$$

$$\frac{2\pi}{T} = \sqrt{\frac{g}{\ell\cos\theta}}$$

$$T = 2\pi\sqrt{\frac{\ell\cos\theta}{g}} \quad (+2)$$

5 b.)



$$x_p = R\cos\theta$$
$$y_p = R\sin\theta \quad (+2)$$
$$\theta = \omega t \quad (+1)$$

$$x_p(t) = R\cos\omega t$$

$$y_p(t) = R\sin\omega t$$

$$\vec{r}_p(t) = R\cos(\omega t)\hat{x} + R\sin(\omega t)\hat{y} \quad (+2)$$

8 c.) $$\vec{v}_p(t) = \frac{d\vec{r}_p}{dt} = -R\omega\sin(\omega t)\hat{x} + R\omega\cos(\omega t)\hat{y} \quad (+3)$$

$$\vec{a}_p(t) = \frac{d\vec{v}_p}{dt} = -R\omega^2\cos(\omega t)\hat{x} - R\omega^2\sin(\omega t)\hat{y} \quad (+3)$$

$$\vec{a}_p(t) = -\omega^2\vec{r}_p(t) \quad (+2)$$