UNIVERSITY OF CALIFORNIA, BERKELEY
MECHANICAL ENGINEERING
ME106 Fluid Mechanics
2nd Test, S10 *Prof S. Morris*

Mean: 144/200
Standard Deviation: 44

1. (65) In a manufacturing process, incompressible viscous polymer is squeezed between a stationary die, and descending mold. As a result, polymer is squeezed axisymmetrically from the gap; all 3 Cartesian components v_x, v_y and v_z of the velocity vector are non–zero. The x and y velocity components are given by the expressions

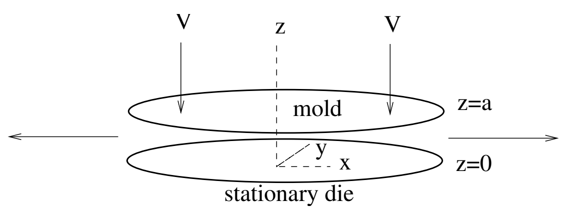
$$v_x = kx(az - z^2), \;\; v_y = ky(az - z^2).$$

Mean: 50/65
Standard Deviation: 18

Using the continuity equation, namely

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

and the boundary conditions on v_z, derive the expression giving v_z as a function of z, a and the (positive) speed V. (The parameter k must **not** enter into your final answer.)



SOLUTION.

Substituting v_x, v_y into the continuity equation, we obtain

$$2k(az - z^2) + \frac{\partial v_z}{\partial z} = 0.$$

+ 25 points

Integrating from $z = 0$ to z, and applying the b.c. $v_z = 0$ on $z = 0$, we find that

+ 10 points

$$-v_z = \tfrac{1}{3}k(3az^2 - 2z^3) \qquad (1)$$

+ 10 points

Applying the remaining b.c. $v_z = -V$ on $z = a$, and solving for k, we obtain

+ 10 points

$$k = 3V/a^3.$$

+ 10 points

Subsituting for k into Eq.(1), and rearranging, we find that

$$v_z = -V\left\{3\left(\frac{z}{a}\right)^2 - 2\left(\frac{z}{a}\right)^3\right\}. \qquad (QED)$$

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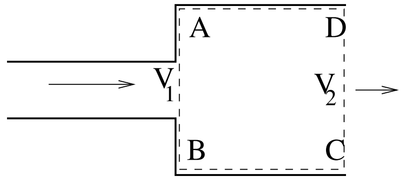
2. (65) Mass conservation requires an incompressible fluid to decelerate as it flows through a sudden expansion in a duct, so that the pressure rises from AB to CD. Assuming that everywhere on AB, $p = p_1$, and that on CD, $p = p_2$, where $p_1 - p_2 = \rho V_2(V_2 - V_1)$, and using the control volume shown, derive the expression giving the mechanical energy loss ΔE per unit mass flowing through the expansion in terms of V_1 and V_2. **Your answer must show explicitly that $\Delta E \geq 0$.**



Mean: 53/65
Standard Deviation: 16

SOLUTION.

Balancing mechanical energy, we find that

$$\dot{m}\left[\tfrac{1}{2}V^2 + \frac{p}{\rho}\right]_1^2 = -\dot{m}\Delta E.$$

+ 30 points
(-5 points if starting from Balance of Total Energy)

(Terms representing shaft power and change in gravitational potential energy vanish identically.) Cancelling $\dot{m}$, and substituting for $p_1 - p_2$, we obtain

$$\tfrac{1}{2}(V_2^2 - V_1^2) - V_2(V_2 - V_1) = -\Delta E.$$

+ 15 points

Rearranging, we obtain the following:

$$\Delta E = \tfrac{1}{2}(V_1 - V_2)^2. \qquad (QED)$$

+ 20 points

The mechanical energy loss ΔE per unit mass is positive, as required by the second law of thermodynamics.

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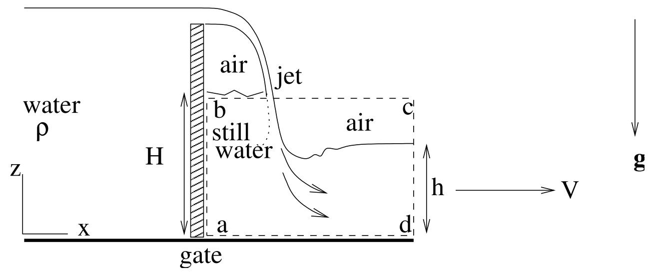
3. (70) Water flowing over a tall sluice gate in a river re-enters the river downstream almost vertically. Because there is a net flow of horizontal momentum out of the control volume illustrated, a horizontal force must act on the material within the control volume. As a result, water backs up to depth H between the gate and the vertical jet; the horizontal force results because $H > h$. Assuming that the water of depth H is stagnant, and that the flow downstream is purely horizontal with speed V, derive the equation giving H in terms of h, V and g.

Mean: 42/70
Standard Deviation: 24



SOLUTION.

+ 20 points

The net flow rate of x–momentum out of control volume $abcd$ is $(\rho V)(Vh)$ (per unit length into the page).

The only forces exerted in the x–direction on matter in the control volume are the pressure forces exerted on surfaces ab and cd.

On surface ab, the pressure at height z above the river bottom is given by $p = p_A + \rho g(H - z)$. The x–component of force acting on a strip $\mathrm{d}z$ is $p\,\mathrm{d}z$, and the resultant horizontal force is given by

$$F_{ab} = \int_0^H p\,\mathrm{d}z, = p_A H + \tfrac{1}{2}\rho g H^2.$$

+ 10 points

On surface cd, the pressure force acts to the left, and is given by $F_{cd} = -\int_0^H p\,\mathrm{d}z$. In calculating the integral, we note that in the air for for $z > h$, $p = p_A$; in the water for $z < h$, $p = p_A + \rho g(h - z)$. Evaluating the integral, we find that

+ 10 points

$$F_{cd} = -p_A H - + \tfrac{1}{2}\rho g h^2.$$

The resultant rightward force is $F_{ab} + F_{cd} = \frac{1}{2}\rho g(H^2 - h^2)$. We note that this is independent of p_A; equally, we could have used the gauge pressure.

The balance of x–momentum is as follows:

+ 30 points

$$\rho V^2 h = \frac{1}{2}\rho g(H^2 - h^2).$$

Solving for H we find that

$$H = h\sqrt{1 + 2\frac{V^2}{gh}} \qquad (QED)$$

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