MT1 7B PROBLEM ①

(a) $\frac{3}{2}K_BT = \frac{1}{2}m_p\overline{U_p^2} \Rightarrow$

$\Rightarrow T = \left(\frac{1}{3}m_p\overline{U_p^2}\right)\cdot\frac{1}{K_B} \Rightarrow$

$T = \frac{1}{3}\cdot 1{,}67\cdot 10^{-27}\,kg\cdot(2\cdot 10^4)^2\frac{m^2}{s^2}\,\frac{1}{1{,}38\cdot 10^{-23}\frac{J}{K}} =$

$= 16.135K = 16{,}14\ {}_{k}K$

(b) $T_e = T_p$ in equlibrium $\Rightarrow$

$m_p\overline{U_p^2} = m_e\overline{U_e^2} \Rightarrow$

$\sqrt{\overline{U_e^2}} = \sqrt{\frac{m_p}{m_e}}\cdot\sqrt{\overline{U_p^2}} = 43{,}1\cdot 20\,\frac{km}{s} \approx 860\,\frac{km}{s}$

(c) $PV = NkT = n\cdot R\cdot T \Rightarrow$

$P = \frac{nRT}{V} = \frac{1}{1m^3}\,2\cdot 10^{-9}_{moles}\,8{,}31\frac{J}{mole\,K}\cdot 16135K =$

$= 2{,}68\cdot 10^{-4}\,Pa$

PROBLEM 2 SOLUTION

Volume changes from V to V' and the temperature changes from T to T'.

a [1pt] $V' = V(1 + \beta\Delta T)$

[1pt] $\Delta V = V\beta\Delta T$

b [2pts] $PV = NkT$

[2pts] $PV' = NkT'$

[2pts] $V' - V = \Delta V = \frac{Nk}{P}\Delta T$

[2pts] $\beta = \frac{\Delta V}{V\Delta T} = \frac{Nk}{PV} = \frac{1}{T}$

PROBLEM 3 SOLUTIONS

Part A

This is a straightforward application of the First Law of Thermodynamics. Remember sign conventions

$$\begin{aligned}\Delta E &= Q - W\\ \Delta E &= (-85J) - (-55J)\\ \Delta E &= -30J\end{aligned}$$

Part B

Since internal energy is a state variable, the change in energy along path *cda* is the opposite of that along path *ac*. Apply First Law of Thermodynamics again

$$\begin{aligned}\Delta E &= Q - W\\ 30J &= Q - 38J\\ Q &= 68J\end{aligned}$$

Part C

Work in both paths *cda* and *abc* take place during isobaric processes. Constant pressure allows us to write

$$W = P\Delta V$$

ΔV for path *cd* is the opposite of that of path *ab*, and we were given work along path *cd* in part B. Because no work is done during the isochoric processes, we can write.

$$\begin{aligned} W_{cd} &= P_d \Delta V_{cd} \\ W_{ab} &= P_a \Delta V_{ab} \\ W_{ab} &= 2.2 P_d (-\Delta V_{cd}) \\ W_{ab} &= -2.2 W_{cd} \\ W_{ab} &= (-2.2)(38J) \\ W_{ab} &= -83.6J \end{aligned}$$

That is, 83.6 J of work is done on the gas.

Problem 4 - 25 points

A hollow, thermally insulated container holds 1 mole of monatomic cas at $0\,^\circ C$. *Some firecrackers in the container explode and suddenly heating the gas to* $50\,^\circ C$. *Beginning with the First Law of Thermodynamics (i.e. not with the explicit expression for the entropy of an ideal gas that we derived in class), calculate the change in entropy of the gas*

Note: Throughout this solution I will note the approximate value of points awarded for a concept. At the end of the solution I include a list of common mistakes. Also, there are several subtleties to this problem, and you were not expected to point them out or fully understand them. Regarding these subtleties, I will point them out and I was very happy that some of you recognized these, but you were not penalized for not recognizing them.

In this problem a gas is rapidly heated inside a container. We assume the container is rigid so that it stays at constant volume when the gas is heated. Thus, since the gas is kept at constant volume, it does no work:

$$V \text{ constant implies } dV = 0 \text{ and therefore } dW = PdV = 0$$

(recognizing this is a constant volume process is worth 5 points). The container is thermally insulated, so no heat flows from the environment into or out of the container: $\Delta Q_{\text{environment}} = 0$. However, the gas is heated by the firecrackers, $\Delta Q_{\text{gas}} > 0$. Combining what we have said, we can write down the First Law of Thermodynamics for the gas:

$$\begin{aligned} dE &= dQ - dW \\ &\rightarrow dW = 0 \text{ since constant volume} \rightarrow \\ &= dQ \end{aligned}$$

For an ideal, monatomic gas (3 degrees of freedom) we have $E = 3/2nRT$ so $dE = 3/2nRdT$ and hence

$$dQ = dE = \frac{3}{2} nRdT$$

(Writing down the First Law and then doing the above is worth $\sim$ 10 points).

We are now prepared to find the change the entropy of the gas. For a quasistatic process, the infinitesmal change in entropy is given by:

$$dS = \frac{dQ}{T}$$

so that the total change in entropy is given by

$$\Delta S = \int dS = \int \frac{dQ}{T}$$

(This was worth 5 points). Plugging in what we have from above:

$$\begin{aligned}\Delta S &= \int \frac{dQ}{T} \\ &= \int_{T_i}^{T_f} \frac{3}{2} nR \frac{dT}{T} \\ &= \frac{3}{2} nR \ln\left(\frac{T_f}{T_i}\right)\end{aligned}$$

Therefore, for one mol of monatomic gas at an initial temperature of $T_i = 0\,^\circ\mathrm{C} = 273\mathrm{K}$ and heated to a final temperature of $T_f = 50\,^\circ\mathrm{C} = 323\mathrm{K}$ at constant volume, the change in entropy is:

$$\boxed{\Delta S = \frac{3}{2} n\ R\ \ln\left(\frac{T_f}{T_i}\right) = \frac{3}{2}(1\text{ mol})\left(8.31\frac{\text{J}}{\text{K mol}}\right)\ln\left(\frac{323\text{ K}}{273\text{ K}}\right) = 2.09\text{ J/K}}$$

(This was worth approximately 5 points). There is something subtle about the above derivation for the change in entropy. The heating of the gas by the exploding firecrackers is not a quasistatic process and therefore $dS \neq dQ/T$ *along the firecracker path.* However, entropy is a state variable which means that it only depends on the initial and final states. So if we can find a quasistatic path that takes us from the initial state to the final state, we can calculate the change in entropy *along the quasistatic path* using $dS = dQ/T$. Then, by the fact that entropy is a state variable and therefore independent of the path taken, the value of ΔS calculated along the quasistatic path also tells the change in entropy for the non-quasistatic process of the firecrackers exploding and heating the gas. Thus in the above when we set $dS = dQ/T$ and integrate, we are implicitly assuming we are taking a quasistatic, constant volume path to the final state. It is absolutely fantastic if you recognized this, but you were not penalized for not recognizing it.

Common mistakes. Many of the mistakes made could have been avoided by thinking about what is physically happening and explaining it in words before writing down any equations. I highly recommend in the future outlining in words what the apparatus and physical process are and what is happening before writing the first equation that pops into your head. Remember, physics is not math, it is physics and therefore requires words and descriptions!

(1) One of the most common mistakes was saying that $\Delta S = \Delta Q/T$. This is not true in general, but only for a constant temperature (isothermal) process:

$$\Delta S = \int \frac{dQ}{T} \neq \frac{1}{T}\int dQ = \frac{\Delta Q}{T}$$

In other words, the temperature may be changing (as in this problem) and therefore is not constant and cannot be brough outside the integral. I took off 5 - 10 points for this mistake.

(2) Suddenly heated does not mean adiabatic. Adiabatic means no heat transfer, and in this problem heat is added to the gas raising its temperature. If you assumed the process is adiabatic ($dQ = 0$) then you would have no net change in entropy: $dS = dQ/T = 0$, but it is physically clear that the gas is getting more disordered by the heating so we should expect a positive entropy change in the gas. The confusion here I believe stems from the word "suddenly". In brief, when we speak about something being sudden and adiabatic we are really talking about two different time scales. Adiabatic processes are quasistatic, *i.e* they are slow compared to the time scale necessary to establish equilibrium (t_{equilib}).

Adiabatic processes also transfer no heat, so they must happen faster than heat can be conducted (t_{heat}) and this is usually what is meant when we say an adiabatic process is sudden. This can be summarized as:

$$\text{Adiabatic: } t_{\text{equilib}} \ll t_{\text{process}} \ll t_{\text{heat}}$$

(3) Many people started with the First Law ($dE = dQ - PdV$ and then said that for an ideal gas we have $PV = nRT$ so $dV = nRdT/P$. This is not true, because in general both the temperature and pressure change, so a change in volume is given by:

$$\begin{aligned} V &= \frac{nRT}{P} \\ \rightarrow dV &= nR\, d\left(\frac{T}{P}\right) \\ &= nR\left(\frac{dT}{P} - \frac{TdP}{P^2}\right) \neq nR\frac{dT}{P} \end{aligned}$$

Note that if you thought about the problem for a second, you would recognize that it is constant volume and therefore $dV = 0$ for this problem and you could have avoided this messiness.

(4) Too many significant figures. Two or three (I allowed up to four) significant figures are okay. If you gave me 5+ significant figures in your final answer, I subtracted a point.

PROBLEM 5

[25] A container with one mole of atomic oxygen at 0 C is brought into thermal contact with a container holding one mole of molecular oxygen (O_2) at 100 C. The containers are thermally isolated from the outside world, and they remain sealed; the gases are not mixed. Note that at $T = 100$ C, the vibrational degrees of freedom of the O_2 are not excited. What is the final temperature if each container is maintained at

- **[12]** constant volume;

  [0] From the notecard:

  $$Q = nC_v\Delta T$$

  A common error made here was using mC_v instead of nC_v; C_v for an ideal gas is already in moles so the masses of the particles need not appear.

  [0] Also from the notecard:

  $$C_v = \frac{d}{2}R$$

  [4] The two containers are thermally isolated from their environment, so the total energy is conserved:

  $$Q_O = -Q_{O_2}$$

  or, more usefully for part b,

  $$C_{v,O}(T_f - T^1_{i,O}) = -C_{v,O_2}(T_f - T^2_{i,O_2})$$

  A common error made here was forgetting the minus sign.

  [4] Correct expression for C_v, including the correct degrees of freedom for each gas:

  $$\frac{3}{2}R(T_f - T_{i,O}) = -\frac{5}{2}R(T_f - T_{i,O_2})$$

[**3**] Solve for T_f:

$$T_f = \frac{1}{8}(5T_{i,O_2} + 3T_{i,O})$$

[**1**] Correct numerical result with correct units:

$$T_f = \frac{1}{8}(5 \times 373\ \mathrm{K} + 3 \times 273\ \mathrm{K}) = 336\ \mathrm{K}$$

Many people gave correct answers working the problem in Celsius; this worked fine in this problem, but in general its always best to convert everything to SI to avoid possible errors.

- [**13**] constant pressure.

  Lazy person's solution: this problem is exactly the same as the prior problem, except the specific heat of the gas should be given at constant pressure instead of at constant volume. Thus:

  [**12**] Realizing you can take your final relation from the last problem and replace C_v with C_p.

  [**1**] Correct numerical answer with correct units.

  Lots of work solution (most people did this):

  [**0**] From the notecard:

  $$Q = nC_p\Delta T$$

  [**0**] Also from the notecard:

  $$C_p = \frac{d+2}{2}R$$

  [**4**] Energy is conserved:

  $$Q_O = -Q_{O_2}$$

  [**4**] Correct expression for C_p, including the correct degrees of freedom for each gas:

  $$\frac{5}{2}R(T_f - T_{i,O}) = -\frac{7}{2}R(T_f - T_{i,O_2})$$

  [**4**] Solve for T_f:

  $$T_f = \frac{1}{12}(7T_{i,O_2} + 5T_{i,O})$$

  [**1**] Correct numerical result with correct units:

  $$T_f = \frac{1}{12}(7 \times 373\ \mathrm{K} + 5 \times 273\ \mathrm{K}) = 331\ \mathrm{K}$$