Midterm 2 Solutions

4/16/2010

1. (a) Answer: ii, iii, iv and v

   See problem set 5 question 3 and PS 6 question 3

   Scoring: $5x - 5y$, where x are the number of correct answers and y are the number of incorrect answers. Note that negative scores were treated as 0.

   (b) Answer: ii, iii, and v

   See PS 6 problem 4 and PS 7 problem 5

   Scoring: $5x - 5y$

   (c) Answer: i, iii, and v

   We are looking at a neutral 2-electron atom. Because the average electron-nuclear separation for the electron in the 4p orbital is so much greater than that of the electron in the 1s orbital we can consider the 1s electron to perfectly shield the nucleus. This gives $E_0/16$ as a crude estimate of the IP.

   Remember that the eigenvalues of $\hat{L}^2$ are $\hbar^2 l(l+1)$. Furthermore the l quantum number for the 1s state is 0 while for the 4p state it is 1.

   We have given no information on how this state was prepared therefore it could be either a singlet or a triplet.

   Scoring: $5x - 5y$

2. (a) Answer: iii, and iv

   Only 1 electron is added to the system, thus the lowest energy state is the symmetric combination of $|A\rangle$ and$|B\rangle$. The normalization constant for this state is,

$$N^2 = (\langle A| + \langle B|)(|A\rangle + |B\rangle) = 2 + 2e^{-R/a}$$

   Which shows iv to be true. Calculating the expectation value of the Hamiltonian using this state (i.e. the energy) gives iii.

   Scoring: $8x - 4y$

(b) Answer: i, iii, and v

When U is negligible the state of the system is best described by the following normalized wave function,

$$|\psi\rangle = \frac{|AA\rangle + |BB\rangle + |AB\rangle + |BA\rangle}{1 + e^{-R/a}}$$

Finding the expectation value of $\mathscr{H}$ with this wave function when U is negligible gives i.

When U is extremely large the state of the system is best described by the following normalized wave function,

$$|\psi\rangle = \frac{|AB\rangle + |BA\rangle}{2 + 2e^{-2R/a}}$$

Finding the expectation value of $\mathscr{H}$ with this wave function gives iii.

Using the first wave function and finding the expectation value of $\mathscr{H}$ when U is neither large or small gives the answer in i plus the answer in v.

Scoring: $6x - 6y$

(c) Answer: ii and iv

The lowest energy spatial wave function in this case is,

$$|\psi\rangle = \frac{|AB\rangle + |BA\rangle}{2 + 2e^{-2R/a}}$$

If both electrons are to occupy this spatial wave function at the same time (which would be necessary for it to be the lowest energy state) then the system must have an anti-symmetric singlet spin wave function. This gives ii.

When U is finite then it is possible to find both electrons around the same nucleus. If this is the case then the spin part of the wave function must be an anti-symmetric singlet.

Scoring: $8x - 8y$

Please check that your scores are calculated correctly.

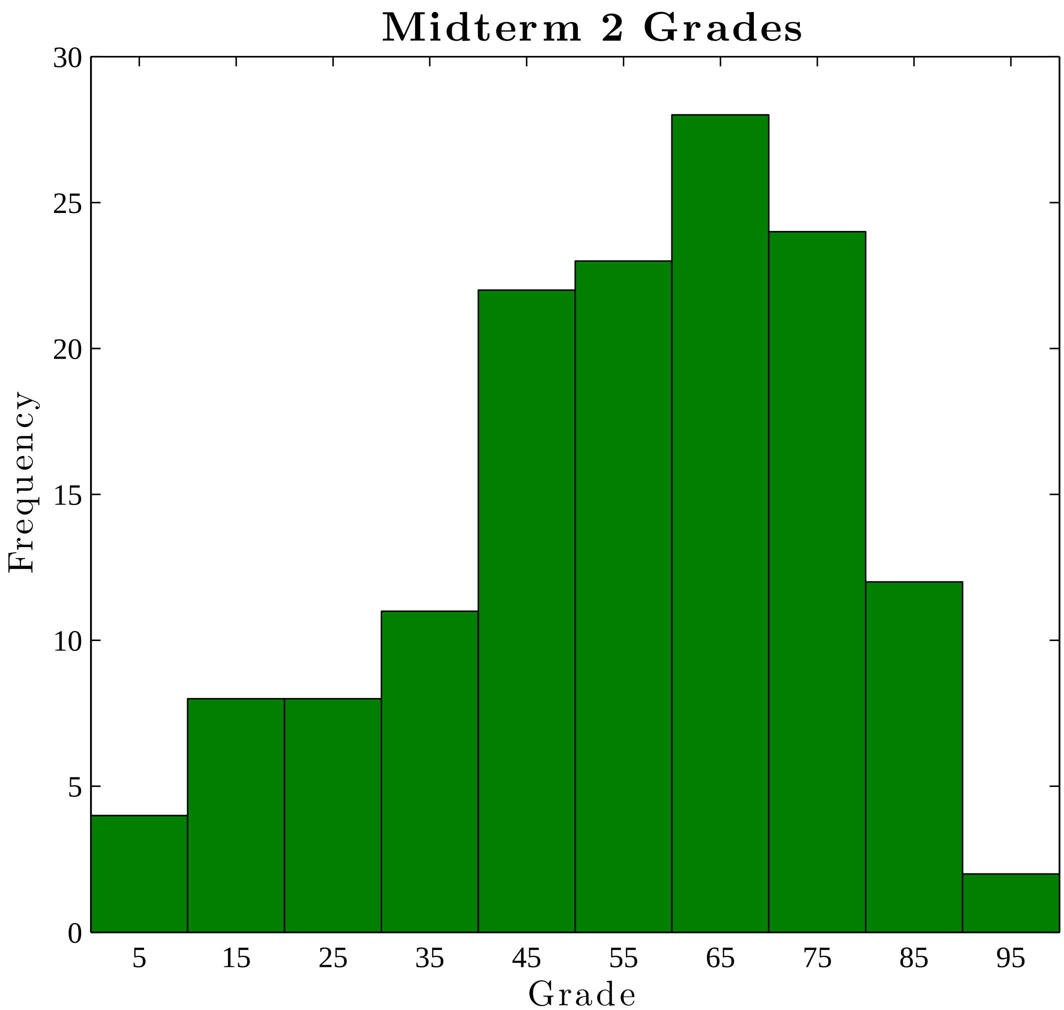

FIGURE 1: **Midterm grade distribution. The mean is 56, median is 59 and the standard deviation is 22. The maximum grade was 100 and the minimum was 0**