Midterm 1 Solutions

2/26/2010

1. (a) Answer: iii and vii

   See problem set 3 question 2.b).

   Scoring: $7x - 3.5y$, where x are the number of correct answers and y are the number of incorrect answers. Note that negative scores were treated as 0.

   (b) Answer: ii and v

   See problem set 3 question 2.b).

   Notice that multiplying a wave function by i does *not* change the wave function.

   Scoring: $7x - 4y$

   (c) Answer: ii, iv, and vi Remember that for any operator we have,

$$\frac{d\langle \hat{G} \rangle}{dt} = \frac{1}{i\hbar}\langle [\hat{G}, \mathscr{H}] \rangle$$

   This equation was given to you at the beginning of the exam. Of course, $\mathscr{H}$ commutes with itself and so its expectation value is time-independent.

   You can calculate the uncertainty in $\mathscr{H}$ which is K.

   Problem set 3 question 2.c) provides the final answer.

   Scoring: $6x - 6y$

   (d) Answer: iii, iv, v, and vii

   $|A\rangle$ and $|B\rangle$ are orthonormal and the action of $\hat{C}$ on both states is given. Using this information you can show that iii and iv are true.

   You know the action of $\hat{C}$ and $\mathscr{H}$ on both $|A\rangle$ and $|B\rangle$ so you can create the matrix representation of both operators in the $|A\rangle$ $|B\rangle$ basis which gives,

$$\mathscr{H} = \begin{pmatrix} \epsilon & -K \\ -K & \epsilon \end{pmatrix}$$

$$\hat{C} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

And you can show that these commute with each other.

Using the matrix form of $\hat{C}$ you can show that vii is true.

Scoring: $4.5x - 4.5y$

2. (a) Answer: i, iv and v

   To be an odd function the function must be antisymmetric about the x axis which means that it must have a node at $x = 0$.

   Using the Virial theorem iv can be shown to be true.

   See problem set 3 question 5 which shows that,

   $$\Delta x = \sqrt{\frac{\hbar\left(n + \frac{1}{2}\right)}{m\omega}}$$

   Thus v is true.

   Scoring: $6x - 5y$

   (b) Answer: ii, v and vi

   You can use the fact that $\langle 1|x|0\rangle = \sqrt{\hbar/2m\omega}e^{i\omega t}$ to show that vi is true.

   Knowing that vi is true shows that ii is true.

   The Virial theorem shows that $\langle 1|x^2|1\rangle = 3\hbar/2m\omega$ and $\langle 0|x^2|0\rangle = \hbar/2m\omega$ while we know that the cross terms are zero because it would be an odd function, $|1\rangle$, times an even function, x^2, times an even function, $|0\rangle$, which is odd overall. See Problem set 4 question 3. So v is true.

   Scoring: $6x - 6y$

Please check that your scores are calculated correctly.

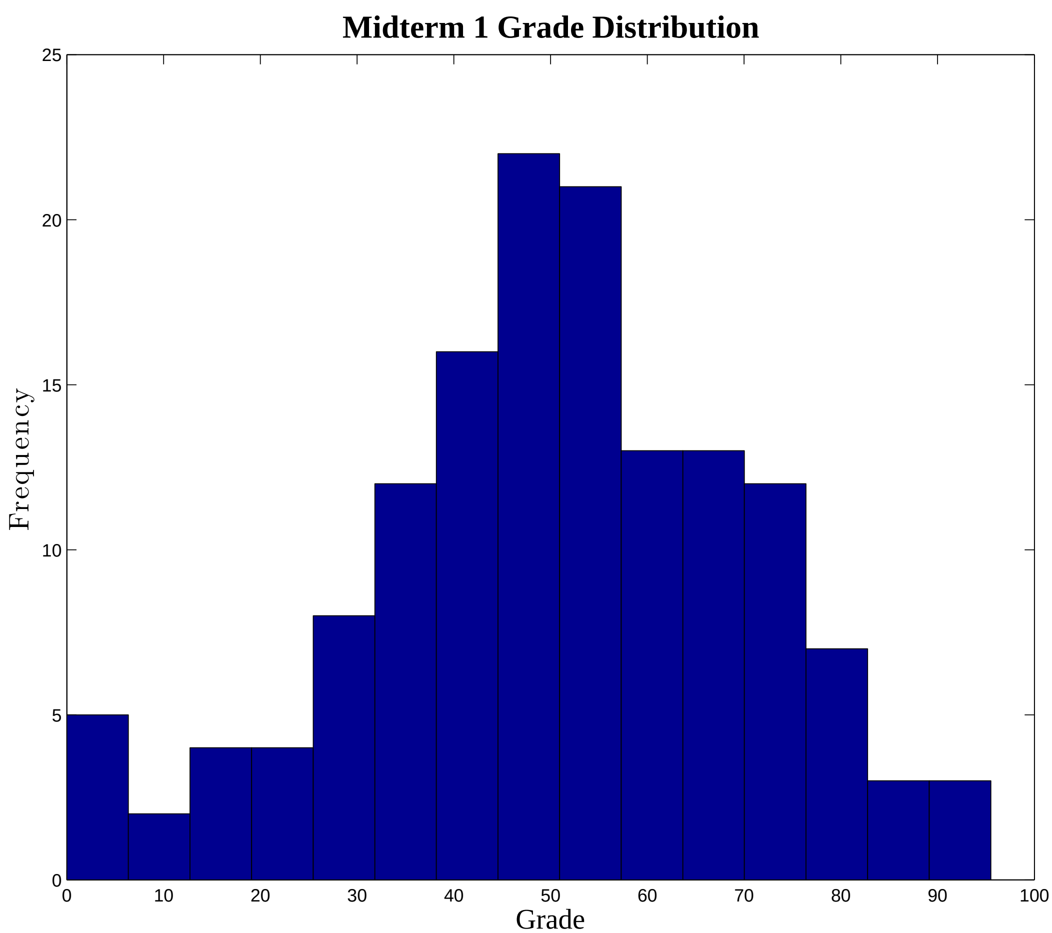

FIGURE 1: **Midterm grade distribution. The mean/median is 50 and the standard deviation is 20.**