CS 172: Computability and Complexity, Spring 2010
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Midterm 1

March 1, 2010

YOUR NAME:

Instructions:

This exam is open-book, open-notes. Please turn off and put away electronic devices such as cell phones, laptops, etc.

You have a total of 75 minutes. There are 4 questions worth a total of 100 points. The questions vary in difficulty, so if you get stuck on any question, it might help to leave it and try another one.

Answer each question in the space provided below the question. If you need more space, you can use the reverse side of that page. *You can use without proof any result proved in class, in Sipser's book, or during discussion sections, but clearly state the result you are using.*

Do not turn this page until the instructor tells you to do so!

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Total	

Problem 1: [True or False, with justification] (30 points)

For each of the following questions, state TRUE or FALSE. Justify your answer in brief, indicating only the "proof idea" or counterexample, drawing a diagram if needed.

(a) If L is regular, then so is any subset of L.

(b) Consider the grammar $G = (\{S, A\}, \{0, 1\}, R, S)$ where R has the rules
$S \rightarrow 0S1 \mid A1$ and $A \rightarrow \epsilon \mid 0S$
Then $L(G)$ contains the string 00111.

(c) The regular expression $(10 \cup 1)1^*$ generates the language
$\{w \mid w \in \{0, 1\}^*$ and w contains at least one 1 and at most one $0\}$.

Problem 2: (25 points)

(a) Consider the language L consisting of all binary strings with an even number of 1s that are not *immediately preceded* by a 0.

For example, $011011 \in L$ because it has two 1s that are not immediately preceded by a 0, whereas $111 \notin L$ because it has three 1s that are not immediately preceded by a 0.

Draw a DFA with *four states* that recognizes L.

(b) Show that any DFA that recognizes L has at least 4 states.

Problem 3: (20 points)

Let $L = \{1^n \mid n \text{ is a prime number}\}$. Prove L is not regular.

[Recall that an integer n is said to be composite (i.e., not prime) if there exists integers $n_1, n_2 > 1$ such that $n = n_1 n_2$.]

Problem 4: (25 points)

A *two-stack pushdown automaton* (2-stack PDA) is exactly like a pushdown automaton except that it has two stacks from which we can push and pop at each step. More formally, a 2-stack PDA consists of a 6-tuple $(Q, \Sigma, \Gamma, \delta, q_0, F)$ where the transition function is defined as

$$\delta : Q \times \Sigma_\epsilon \times \Gamma_\epsilon \times \Gamma_\epsilon \rightarrow 2^{(Q \times \Gamma_\epsilon \times \Gamma_\epsilon)}$$

If $(q', b_1', b_2') \in (q, a, b_1, b_2)$, it means that the 2-stack PDA can read the input character a, pop b_1 from the first stack, pop b_2 from the second stack, push b_1' onto the stack, push b_2' onto the stack, and go from state q to state q'. The acceptance condition for a 2-stack PDA is just as in the PDA.

(a) Let L_1 and L_2 be context-free languages. Show that $L_1 \cap L_2$ is recognizable by a 2-stack automaton.

(b) Show that a 2-stack PDA is more powerful than a PDA (i.e. it can recognize more languages) by showing that a 2-stack PDA exists that recognizes the language $L = \{0^n 1^n 2^n \mid n \geq 0\}$.