Mathematics 54
Midterm #1 Spring 2006
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Instructions Write your name, section number, and GSI's name on your Blue Book RIGHT NOW. Show your work on Problems 2-4. Partial credit may be given, but only if the work justifies it. Best wishes on the exam !

Problem #1 Answer True or False in your Blue Book.

(A) The following matrix is in row echelon form.

$$\begin{pmatrix} 1 & -1 & 1 & 0 & 6 \\ 0 & 0 & 1 & -9 & 2 \\ 0 & 0 & 0 & 1 & 10 \end{pmatrix}$$

(B) The set consisting of those 3-tuples (x, y, z) satisfying the equation $x + 2y + 3z = 0$ is a subspace of R^3 .

(C) The set of polynomials $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ such that $p'(0) = 0$ is a subspace of P_3 . Here $p'(x)$ denotes the derivative of $p(x)$.

(D) The subset of R^4 consisting of all points of the form $\{t, t^3, 0, 2t\}$ where t is any real number is a subspace of R^4.

(E) Any 2×2 matrix A satisfying the equation $A^2 + A - 5I = 0$ is invertible.

(F) The angle between the two vectors $u = (1, 2, -1, 1)$ and $v = (3, -1, 8, 7)$ is 90 degrees.

(G) Let A and B be two 5×5 matrices . Then $Trace(3A + B) = 3Trace(A) + Trace(B)$

(H) Let A be a 2×2 matrix . Then $Det(3A) = 3Det(A)$.

(I) The functions $f(t) = 1$, $g(t) = cos^2(t)$, and $h(t) = sin^2(t)$ are linearly dependent when considered as elements in the vector space $C[0, 1]$.

(J) $E_{21}(a)E_{21}(b) = E_{21}(a + b)$

$\Longrightarrow$

Problem #2 Consider the vectors $v_1 = (1, 0, 1)$, $v_2 = (1, 1, 2)$, and $v_3 = (1, -1, 0)$. Let $b = (4, 0, 0)$.

(A) Determine whether the vectors $\{v_1, v_2, v_3\}$ are linearly independent.

(B) Write down explicitly the linear equations which must be solved in order for b to belong to $Span\{v_1, v_2, v_3\}$.

(C) Now determine whether b is actually in $Span\{v_1, v_2, v_3\}$ by using row operations on the appropriate augmented matrix to transform it to row echelon form. List in order the row operations (i.e., the elementary matrices) you use during this process.

Problem #3 **(A)** Consider the matrix

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix}$$

$\checkmark$ $E_2 E_1 = A^{-1}$

Find A^{-1} using the method of row operations. In particular, list in order the row operations(i.e., the elementary matrices) you use during this process.

(B) Suppose A and B are 2×2 matrices with $det(A) \neq 0$. Determine whether there is always a 2×2 matrix C satisfying the equation $AC = B$? If so, give a formula for C. If not, give an example where there is no such C.

Problem #4 Let P_2 be the vector space of all polynomials $p(x) = a_0 + a_1 x + a_2 x^2$. Consider the inner product on P_2 defined by the formula

$$\langle f(x), g(x) \rangle = \int_{-1}^{1} f(x) g(x) dx$$

(A) Determine whether the polynomials $f(x) = 1$ and $g(x) = x$ are orthogonal (i.e., perpendicular) .

(B) Let $\{v_1, v_2, v_3\}$ be a set of linearly independent vectors in a vector space. Write down the Gram-Schmidt equations for obtaining a set $\{w_1, w_2, w_3\}$ of vectors which are orthogonal to each other and where $Span\{v_1, v_2, v_3\} = Span\{w_1, w_2, w_3\}$.

(C) Now apply the Gram-Schmidt process to the three polynomials $f(x) = 1, g(x) = x, h(x) = x^2$.