Solutions to the IEOR 130 Midterm Examination
Spring, 2010
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Open books and notes. Work all problems. 20 points each problem, 60 points total.

1. A fabrication plant includes a sophisticated etching machine purchased almost a year ago. The purchase agreement for the machine included a service contract lasting one year whereby technicians working for the machine vendor perform preventative maintenance and repairs on the machine. At present, a machine vendor's technician performs a weekly PM. The PM takes the machine down for 4 hours. When the machine breaks down, the down time averages 8 hours (including time for the technician to drive to the plant). Data on machine failures indicates that the time until failure from performance of PM is distributed as follows:

Days since PM, *t*	Fraction of breakdowns occurring on day *t*
1	0.04
2	0.04
3	0.05
4	0.05
5	0.06
6	0.06
7	0.07
8	0.07
9	0.08
10	0.08
11	0.09
12	0.09
13	0.10
14	0.12

The service contract is about to expire. The machine vendor offers to renew the service contract for one year at a fixed cost of $150,000. Alternatively, the plant could hire a local, on-call independent contractor charging $250 per hour to perform PMs or repairs. This contractor used to work for the vendor and is very knowledgeable about the machine. It is believed that the contractor could perform high-quality maintenance work just as quickly as the vendor's staff.

(a) (5 points) The machine vendor is currently performing weekly PMs. Estimate the availability of the machine.

The expected down time per cycle, in hours, is $c_1F(t) + c_2[1-F(t)]$ where $c_1 = 8$ hours, $c_2 = 4$ hours, and t = 7 days. For the given data, F(7) = 0.37, so the expected downtime per cycle is 8*0.37 + 4*0.63 = 5.48 hours.

The expected length of a cycle is Σ k*pk + t*[1-F(t)], which for the given data, is 1*0.04 + 2*0.04 + 3*0.05 + 4*0.05 + 5*0.06 + 6*0.06 + 7*0.07 + 7*0.63 = 6.03 days.

The expected availability is thus 1 – 5.48/24*6.03 = 96.21%.

(b) (5 points) What frequency of PMs would you recommend to maximize machine availability?

Let G(t) = expected non-availability of the machine if PMs are scheduled every t days. IN part (a), we calculated G(7). If one calculates, G(1), G(2), …, G14), then G(10) = 3.463% is the optimum, and scheduling a PM once every 10 days provides the maximum expected availability.

(c) (5 points) Estimate the availability if the frequency of PM was changed to follow your recommendation in (b).

1 – 0.03463 = 96.54%.

(d) (5 points) Estimate the annual costs for maintenance of the machine if the plant terminates the service contract and instead utilizes the local independent contractor following the PM frequency you calculated in (b). Would you recommend renewing the service contract? Or hiring the local contractor?

Now let's use c_1 = 8*250 = $2,000 and c_2 = 4*250 = $1,000 to find the expected cost rate. The expected cost/day is G(10) = ($2,000*0.6 + $1,000*0.4) /[Σ k*pk + 10*(1-F(10))] = $207.79/day or $75,844 per year. So it seems that it would be wise to terminate the service contract and utilize the on-call local contractor.

2. Two factories *A* and *B* make the same product in three manufacturing steps. Each step has an upper specification limit but no lower specification limit. Data on the process performance index (C_{pk}) for each of the steps at each factory is as follows:

Step	Fab A	Fab B
1	0.75	0.70
2	0.75	0.65
3	0.65	0.80

Suppose the only yield loss mechanisms are from exceeding the upper spec limits. Further, suppose yield losses at each step are independent. Assume there is 100% inspection after each step, and bad units are discarded before processing by the next step.

(a) (5 points) Explain why, for any of the steps above, the yield of the step may be well-estimated as Prob{$Z < 3*C_{pk}$} where Z is ~ N(0,1).

Cpk = (USL – m)/3s, or USL = m + 3*Cpk*s. So Yield = Prob { X < m + 3*Cpk*s }. Suppose we assume X is normally distributed. Then Yield = Prob { Z < 3*Cpk } where Z ~ N(0,1). Assuming Cpk is reasonably large, then yield is expressed as an integral of most of its probability mass, except for a small portion of the tail of the distribution. In that case, the normal approximation will be a pretty good one.

(b) (5 points) Estimate the overall yield at each factory. Which factory is doing better?

Step	Factory A	Factory B
1	Y_1 = Prob {Z < 3*0.75} = 0.9878	Y_1 = Prob {Z < 3*0.70} = 0.9821
2	Y_2 = Prob {Z < 3*0.75} = 0.9878	Y_2 = Prob {Z < 3*0.65} = 0.9744
3	Y_3 = Prob {Z < 3*0.65} = 0.9744	Y_3 = Prob {Z < 3*0.80} = 0.9918

Y_A = (0.9878)(0.9878)(0.9744) = 0.9507
Y_B = (0.9821)(0.9744)(0.9918) = 0.9491

It is very close. Factory A is doing slightly better.

(c) (5 points) Suppose the first step involves a countable parameter of quality, and suppose USL for this step is 100. What is the upper control limit of an SPC chart for the first step in Fab A?

For a countable parameter, σ = SQRT(μ). So Cpk = (USL – μ)/3*SQRT(μ). Then

3*0.75*SQRT(μ) = 100 – μ

$5.0625\mu = 10{,}000 - 200\mu + \mu^2$

$0 = 10{,}000 - 205.0625\mu + \mu^2$

Applying the quadratic formula, we find $\mu = 79.89$.

Hence UCL = μ + 3*SQRT(μ) = 79.89 + 26.81 = 106.70.

(d) (5 points) Suppose we could utilize the best step from each fab to make the product. How much better would the yield be?

Y = (0.9878)(0.9878)(0.9918) = 0.9677, almost 1.8 points higher than the average of the two factories.

3. The management of a factory is trying to sort out how much yield loss is coming from a stationary baseline distribution of defects vs. how much is coming from defect excursions and other systematic mechanisms of yield loss. A stacked wafer map is analyzed including only wafers believed to not be involved in any defect excursions. The best-yielding die site on the wafer map has a 65% yield. The number of 0.5 sq-cm dice printed on the wafer is 450. The average die yield over all wafers (including those involved in excursions) is 35%.

(a) (7 points) Calculate the baseline defect-limited yield and the underlying baseline defect density.

Ymax = Yr + 3*SQRT[Yr(1-Yr)/GD]

0.65 = Yr + 3*SQRT[Yr(1-Yr)/450]

Solving for Yr, we find Yr = 58.02%. The equivalent baseline defect density is D = -ln(Yr)/A = 1.089.

(b) (7 points) Management is considering an upgrade of the air flow system costing $1.5 million. Engineering tests have been performed that indicate that this upgrade can be expected to cut baseline particle contamination on the wafers by 20%. However, particle excursions do not seem to be abated by the improved air flow. Estimate the improvement in baseline defect-limited yield and in the overall die yield if this upgrade is undertaken.

D would be reduced to 0.8*1.089 = 0.8710

So Yr would be increased to Yr = EXP(-0.8710*0.5) = 64.69%

Using the current Yr, Ys = Y/Yr = 0.35/0.5802 = 60.32%

The overall die yield would rise to Y new = (0.6032)(0.6649) = 39.03%

(c) (6 points) Management also is considering investment in a $1.5 million inspection system enabling increased process monitoring so that excursions can be detected earlier and thereby reduce yield losses. Engineering analysis and experiments indicate that total systematic and excursion yield losses could be cut 20% by this investment. Assuming the air flow system is NOT upgraded, what overall die yield would result from implementation of this inspection system? If only $1.5 million is available to spend, which is a better expenditure for improving yield – the air flow system upgrade, or the new inspection system?

Ys would be improved to 0.2*1 + 0.8*0.6032 = 0.6826

The overall die yield would rise to Y new = (0.6826)(0.5802) = 39.60%

From the point of view of yield, the inspection system would seem to be a slightly better investment than the upgrade to the airflow system.