Department of Mechanical Engineering
University of California at Berkeley
ME 104 Engineering Mechanics II
Spring Semester 2010

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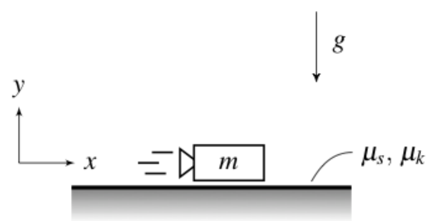
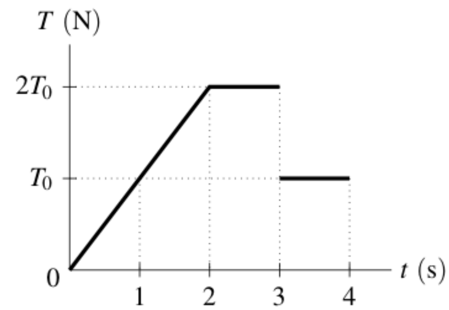
Midterm Examination No. 2 April 2, 2010

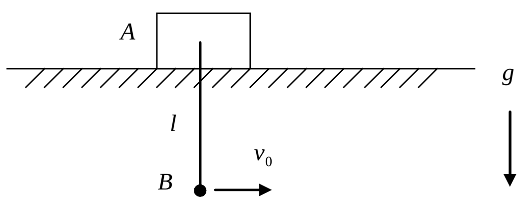
The examination has a duration of 50 minutes.
Answer all questions.
All questions carry the same weight.

1. A block with mass $m = 4\,\text{kg}$ is initially at rest at time $t = 0$ on a rough horizontal surface with coefficients of static and kinetic friction given by $\mu_s = 0.5$ and $\mu_k = 0.25$, respectively. A small booster attached to the block ignites at $t = 0$ and generates a variable thrust $T(t)$ for 4 s, as illustrated below. Let $T_0 = 20\,\text{N}$ and gravitational acceleration $g = 10\,\text{m/s}^2$.
(a) Draw a free-body diagram for the block.
(b) When does the block begin to move?
(c) How fast is the block moving at $t = 4\,\text{s}$?

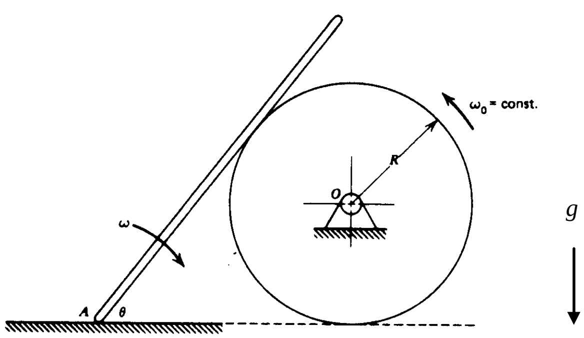




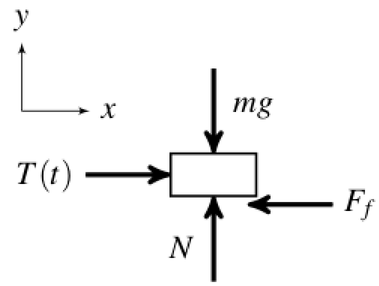
2. Ball *B*, of mass m_B, is suspended from a cord of length l attached to cart *A*, of mass m_A, which can roll freely on a frictionless horizontal track. If the ball is given an initial horizontal velocity v_0 while the cart is at rest, determine (a) the velocity of *B* as it reaches its maximum elevation, and (b) the maximum vertical distance h through which *B* will rise. It is assumed that $v_0^2 < 2gl$.



3. The slender rod rolls without slipping on a circular disk which has a constant angular velocity ω_0. End *A* is constrained to move on a smooth horizontal surface as θ decreases. Determine the angular velocity ω of the rod in terms of ω_0 when $\theta = 70°$.



1. (a) The free-body diagram for the block is given by



(b) Balancing forces in the horizontal and vertical directions,

$$\Sigma F_x = T(t) - F_f = m\ddot{x}$$

$$\Sigma F_y = N - mg = 0 \qquad \Rightarrow \qquad N = mg$$

The block is initially at rest and moves only after it overcomes static friction. The maximum static friction force acting on the block just before it slips is $F_{\max} = \mu_s N = \mu_s mg$. With $\ddot{x} = 0$, the corresponding thrust is

$$T^* = \mu_s mg = (0.5)(4\text{ kg})(10\text{ m/s}^2) = 20\text{ N} = T_0$$

From the given illustration of the thrust over time, it follows that the block starts to move at $t = 1$ s.

(c) When the block starts to slip at $t = 1$ s, the friction force acting on it is $F_f = \mu_k N = \mu_k mg$. By a linear impulse-momentum analysis in the horizontal direction,

$$mv = \int_1^4 \Sigma F_x dt = \int_1^4 (T(t) - \mu_k mg)dt = \int_1^4 T(t)dt - 3\mu_k mg$$

The impulse $\int_1^4 T(t)dt$ attributed to the thrust is given by the area under the illustrated thrust profile from 1 s to 4 s:

$$\int_1^4 T(t)dt = T_0(4-1) + \frac{1}{2}(2T_0 - T_0)(2-1) + (2T_0 - T_0)(3-2) = \frac{9}{2}T_0$$

Therefore, the block's speed v at $t = 4$ s is

$$v = \frac{9T_0}{2m} - 3\mu_k g = \frac{9(20\text{ N})}{2(4\text{ kg})} - 3(0.25)(10\text{ m/s}^2)$$

$$\Rightarrow \qquad v = 15\text{ m/s}$$

2. (a) When ball B reaches its maximum elevation in position 2, its velocity $(\mathbf{v}_{B/A})_2$ relative to cart A is zero. Since A is translating horizontally,

$$(\mathbf{v}_B)_2 = (\mathbf{v}_A)_2 + (\mathbf{v}_{B/A})_2 = (\mathbf{v}_A)_2$$

By conservation of system linear momentum,

$$\Delta G_x = 0 \qquad \Rightarrow \qquad m_B v_0 = m_A(v_A)_2 + m_B(v_B)_2 = (m_A + m_B)(v_B)_2$$

$$\Rightarrow \qquad (v_B)_2 = \frac{m_B}{m_A + m_B} v_0$$

(b) The energy of the system is conserved.

$$\Delta T + \Delta V_g = 0$$

$$\Rightarrow \qquad \frac{1}{2}m_A(v_A)_2^2 + \frac{1}{2}m_B(v_B)_2^2 - \frac{1}{2}m_B v_0^2 + m_A gl + m_B gh - m_A gl = 0$$

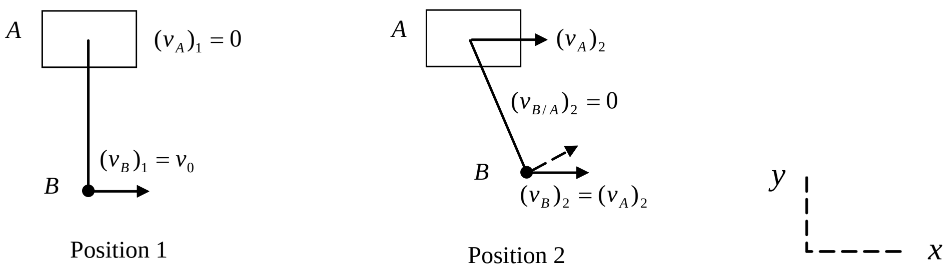
$$\Rightarrow \qquad h = \frac{v_0^2}{2g} - \frac{m_A + m_B}{m_B}\frac{(v_B)_2^2}{2g}$$

Using the result in part (a) for $(v_B)_2$,

$$h = \frac{m_A}{m_A + m_B}\frac{v_0^2}{2g}$$

The assumption $v_0^2 < 2gl$ ensures that

$$h = \frac{m_A}{m_A + m_B}\frac{v_0^2}{2g} < \frac{m_A l}{m_A + m_B} < l$$



3. Since the rod *AD* rolls on the disk without slipping,

$$v_B = R\omega_0$$

In position *θ*, $\mathbf{v}_A$ is horizontal and $\mathbf{v}_B$ is tangent to the disk. Thus the instantaneous center of zero velocity of the rod *AD* is located at *C*, where *CA* is perpendicular to $\mathbf{v}_A$ and *CB* is perpendicular to $\mathbf{v}_B$. In triangle *OAB*,

$$\tan\frac{\theta}{2} = \frac{R}{AB}$$

In triangle *CAB*,

$$\tan\theta = \frac{AB}{CB}$$

$$\Rightarrow \qquad CB = \frac{AB}{\tan\theta} = \frac{R}{\tan(\theta/2)\tan\theta}$$

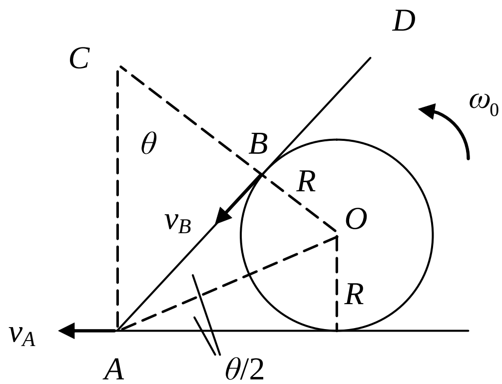
As a consequence,

$$v_B = CB\omega_{AD}$$

$$\Rightarrow \qquad \omega = \omega_{AD} = \frac{v_B}{CB} = \frac{R\omega_0}{R}\tan(\theta/2)\tan\theta = \omega_0\tan(\theta/2)\tan\theta$$

When $\theta = 70°$,

$$\omega = \omega_0 \tan 35° \tan 70° = 1.92\omega_0$$



D
C
ω_0
θ
B
R
v_B
O
R
v_A
A
$\theta/2$