Department of Mechanical Engineering
University of California at Berkeley
ME 104 Engineering Mechanics II
Spring Semester 2010

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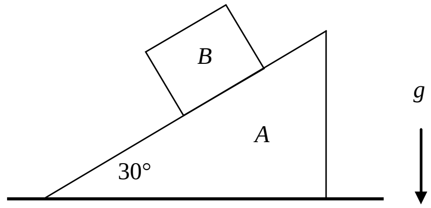
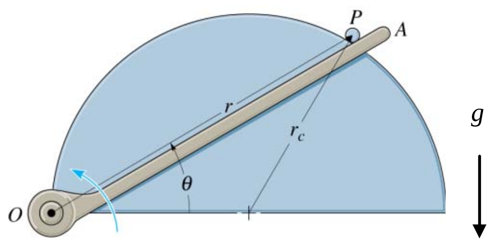
Midterm Examination No. 1 Feb 26, 2010

The examination has a duration of 50 minutes.
Answer all questions.
All questions carry the same weight.

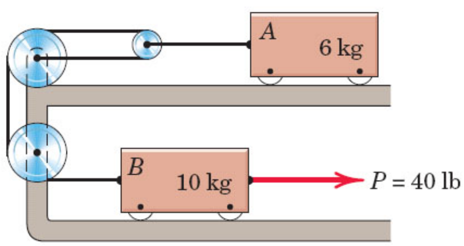
1. The 5-kg block *B* starts from rest and slides on the 10-kg wedge *A*, which rests on a horizontal surface. Neglecting friction, determine the acceleration of the wedge and the acceleration of the block relative to the wedge.



2. A particle *P* of mass *m* is guided along a smooth circular path of radius r_c by the rotating arm *OA*. If the arm has a constant angular velocity ω, determine the angle $\theta \le 45°$ at which the particle leaves the circular path. Some formulas that may be useful are: $a_t = \dot{v}$; $a_n = v^2 / \rho$; $a_r = \ddot{r} - r\dot{\theta}^2$; $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$.



3. The force $P = 40\,\text{N}$ is applied to the system, which is initially at rest. Determine the speeds of *A* and *B* after *A* has moved 0.4 m.



Problem 1. The wedge *A* is in rectilinear motion in an absolute *XY*-frame. Attach *xy*-frame to the wedge, with the *x*-axis directed up the incline. The block *B* is in rectilinear motion in the translating *xy*-frame. Thus

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A} = (a_A \cos 30° - a_{B/A})\mathbf{i} - a_A \sin 30° \mathbf{j}$$

$$\Rightarrow \qquad (a_B)_x = a_A \cos 30° - a_{B/A}, \qquad (a_B)_y = -a_A \sin 30°$$

For wedge *A*,

$$\sum F_X = ma_X \quad \Rightarrow \quad N \sin 30° = m_A a_A$$

$$\Rightarrow \quad N \sin 30° = 10 a_A \qquad (1)$$

For block *B*,

$$\sum F_x = m(a_B)_x \quad \Rightarrow \quad -m_B g \sin 30° = m_B (a_A \cos 30° - a_{B/A})$$

$$\Rightarrow \quad a_{B/A} = a_A \cos 30° + g \sin 30° \qquad (2)$$

$$\sum F_y = m(a_B)_y \quad \Rightarrow \quad N - m_B g \cos 30° = -m_B a_A \sin 30°$$

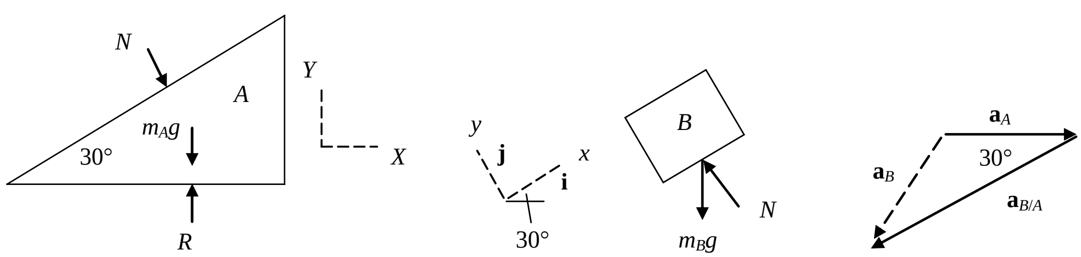
$$\Rightarrow \quad N - 5g \cos 30° = -5a_A \sin 30° \qquad (3)$$

There are two unknowns N, a_A in Eqs. (1) and (3). Eliminate N from these two equations

$$\Rightarrow \qquad a_A = \frac{5g \cos 30°}{20 + 5\sin 30°} = 1.89 \text{ m/s}^2 \qquad \longrightarrow$$

Substitute the value of a_A into Eq. (2),

$$a_{B/A} = 1.89 \cos 30° + g \sin 30° = 6.54 \text{ m/s}^2 \qquad \swarrow$$



Problem 2. Attach *rθ*-frame at *O* with the horizontal as the reference line. The particle is not in circular motion with respect to polar coordinates located at *O*. Suppose the particle leaves the circle of radius r_c at $\beta \le 45°$. At any position $\theta < \beta$,

$$\dot{\theta} = \omega = \text{contant} \quad \Rightarrow \quad \ddot{\theta} = 0$$

$$r = 2r_c \cos\theta \quad \Rightarrow \quad \dot{r} = -2r_c \dot{\theta} \sin\theta = -2r_c \omega \sin\theta$$

$$\Rightarrow \quad \ddot{r} = -2r_c \omega^2 \cos\theta$$

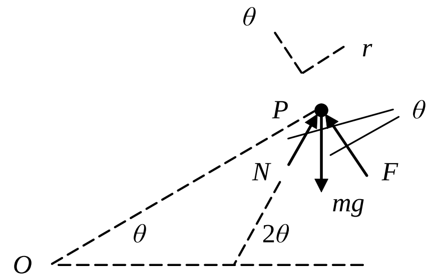
Since the force *F* exerted by the arm *OA* on mass *m* is perpendicular to *OA* while the reaction *N* is normal to the circular path,

$$\sum F_r = ma_r \quad \Rightarrow \quad -mg \sin\theta + N \cos\theta = m(\ddot{r} - r\dot{\theta}^2) = m(-4r_c \omega^2 \cos\theta)$$

When *m* leaves the path at $\theta = \beta$, $N = 0$. Thus from the above equation,

$$-mg \sin\beta = -4mr_c \omega^2 \cos\beta$$

$$\Rightarrow \qquad \beta = \tan^{-1}\left(\frac{4r_c \omega^2}{g}\right)$$



Problem 3. Blocks *A* and *B* perform rectilinear motion. From a vertical reference line through the centers of the pulleys, measure the positions of *A*, *B* by x_A and x_B.

$$2x_A + x_B = \text{constant}$$

$$\Rightarrow \quad 2v_A + v_B = 0$$

Let configuration 1 denote the initial rest positions of blocks *A* and *B*. Suppose configuration 2 corresponds to new positions after *A* has moved by 0.4 m. For the system consisting of *A* and *B*,

$$U = \Delta T = T_2 - T_1 = T_2$$

$$\Rightarrow \quad P\Delta x_B = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$$

$$\Rightarrow \quad 40(0.8) = \frac{1}{2}(6)v_A^2 + \frac{1}{2}(10)(-2v_A)^2$$

$$\Rightarrow \quad v_A = 1.18\,\text{m/s}$$

In addition,

$$v_B = 2v_A = 2.36\,\text{m/s}$$