

Operations Research II, IEOR161
University of California, Berkeley
Midterm II, 2010

1. [15+15+10] Consider a queueing system consisting of 2 servers. Customers arrive according to a Poisson process with rate $\lambda = 4$ (per hour). An arriving customer requests server 1 with probability $p = 0.3$ and with probability $1 - p = 0.7$ requests server 2. Service times of both servers are exponential with rate $\mu = 3$. Each server works on one customer at a time and new customers wait in line for service if their server is busy.
 - (a) Suppose there is one customer at each service desk. What is the expected time for both of these customers to be cleared?
 - (b) Suppose that server 1 is busy. What is the probability that the customer being served by server 1 is cleared from the system before another customer for server 1 arrives?
 - (c) Suppose both servers start idle. After 10 hours, we are told that 15 customers went to server 2 of which 5 still remain in the system. What is the expected number of customers for server 1 to arrive during this (10 hour) period?

2. [10+10]
 - (a) Arrivals occur according to a Poisson process with rate λ . 15 arrivals occur in a 10 hour period. What is the probability that they all occur in the last hour?
 - (b) Arrivals occur according to a Poisson process with rate λ . There are 3 servers in parallel where service times for all servers are exponential with rate μ . Suppose that 2 arrivals occur within 1 hour. What is the probability that both are still in the system at the end of the hour?

3. [15+15+10] Consider a queueing system consisting of 2 servers in parallel where arrivals occur according to a Poisson process with rate λ and service times are exponential with rate μ at both servers. An arrival immediately enters service if one of the servers is free; if both servers are busy the customer leaves (and never returns). Consider this system at "event times" (i.e. arrivals of a new entering customer or departures after a service completion, etc):
 - (a) Write down the transition probabilities when the system state is the number of customers in the system?
 - (b) Write down the balance equations.
 - (c) Calculate the stationary distribution.